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Asymptotic homogenization of hygro-thermo-mechanical properties of fibrous networks

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ABSTRACT

The hygro-thermo-expansive response of fibrous networks involves deformation phenomena at multiple length scales. The moisture or temperature induced expansion of individual fibres is transmitted in the network through the inter-fibre bonds; particularly in the case of anisotropic fibres, this substantially influences the resulting overall deformation. This paper presents a methodology to predict the effective properties of bonded fibrous networks. The distinctive features of the work are i) the focus on the hygro-thermo-mechanical response, whereas in the literature generally only the mechanical properties are addressed; ii) the adoption of asymptotic homogenization to model fibrous networks. Asymptotic homogenization is an efficient and versatile multi-scale technique that allows to obtain within a rigorous setting the effective material response, even for complex micro-structural geometries. The fibrous networks considered in this investigation are generated by random deposition of the fibres within a planar region according to an orientation probability density function. Most of the available network descriptions model the fibres essentially as uni-axial elements, thereby not explicitly considering the role of the bonds. In this paper, the fibres are treated as two dimensional ribbon-like elements; this allows to naturally include the contribution of the bonding regions to the effective expansion. The efficacy of the proposed study is illustrated by investigating the effective response for several network realizations, incorporating the influence of different micro-scale parameters, such as fibre hygro-thermo-elastic properties, orientation, geometry, areal coverage.

1. Introduction

Fibrous networks are encountered in many different engineering applications, ranging from bio-inspired, paper-like to non-woven materials. These materials have an intrinsic multi-scale nature: the constitutive behaviour of the single fibres, the mutual fibres interactions as well as their geometrical features and those of the network all contribute to the effective material response. This work is motivated by paper-based fibrous networks, which are characterized by a strong sensitivity to environmental conditions, such as moisture variations, triggering coupling between their mechanical and hygroscopic response. This is caused by the fact that the hydrophilic paper fibres exhibit large deformations upon humidity changes -up to 20%. The highly anisotropic swelling of individual fibres and the competition between hygro-mechanical properties taking place in the bonding areas strongly affect the effective paper response (Niskanen, 1998; Larsson and Wagberg, 2008). Focus is here put on the hygro-mechanical properties of paper networks. Note, however, that the approach proposed in this work is general as it allows to establish a scale transition between the level of the underlying micro-structure and the effective hygro-mechanical *and* thermo-mechanical (possibly also hygro-thermo-mechanical) properties of a wider class of fibrous materials. This constitutes a significant step forward towards the development of predictive multi-scale models that are indispensable for many applications.

Extensive work has been done in the literature on the prediction of the effective response of fibrous materials. Analytical models (Cox, 1952; Astrom et al., 1994; Wu and Dzenis, 2005; Tsarouchas and Markaki, 2011) are often based on the assumption of affine deformation of the network, i.e. local strains are equal to the macroscopic deformation. They provide closed form solutions of the effective properties as a function of several micro-structural parameters, for instance the fibre and network geometry, distribution, orientation. These models provide direct insight on the effective behaviour of fibrous materials. However, the approximation of affine deformation is not always realistic, as illustrated e.g. in

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Hatami-Marbini and Picu (2009). For this reason, more recently, several numerical or computational models of fibrous networks have been developed (Wilhelm and Frey, 2003; Bronkhorst, 2003; Hagglund and Isaksson, 2008; Hatami-Marbini and Picu, 2009; Kulachenko and Uesaka, 2012; Shahsavari and Picu, 2013; Lu et al., 2014; Dirrenberger et al., 2014; Chen et al., 2016; Ban et al., 2016; Karakoc et al., 2017). Most of these models are based on a two dimensional description of the material, in which the individual fibres are modelled as a series of trusses or beams with generally isotropic constitutive properties. The fibres are connected to each other at the nodes through inter-fibre bonds, whose role is often not studied explicitly. The main focus of all of these descriptions is on the network's mechanical behaviour, possibly including fracture and damage. Despite the relevance of hygro (thermal) effects on fibrous networks, to the best of our knowledge, the literature lacks contributions specifically dedicated to the analysis of their hygro(thermo)-mechanical response.

The objective of the present study is to investigate the effective hygro-mechanical properties of paper-like fibrous materials based on the analysis of the underlying network, using an asymptotic homogenization approach. Asymptotic homogenization (Sanchez Palencia, 1980; Bakhvalov and Panasenko, 1989; Guedes and Kikuchi, 1990) is a multi-scale technique to model heterogeneous materials with a periodic micro-structure. The solution of the equilibrium problem is written as an asymptotic expansion. Inserting this expression into the equilibrium equation leads to an explicit dependence of the displacement (or strain or stress) field on the macro-scale and the micro-scale contributions. This allows to obtain closed form relations for the effective material properties, which are resolved numerically in relation to a given micro-structural geometry.

The micro-structural domain is represented here by a periodic repetition of a network of fibres. Note that the condition of periodicity of the micro-structure is generally not satisfied in real fibrous materials. This issue is circumvented by assuming as a unit-cell a representative volume element, which is large enough to provide sufficient micro-structural information to be representative of a real, disordered micro-scale domain (Drugan and Willis, 1996). The network is generated by depositing the fibres in random positions in a two dimensional region with an anisotropic orientation distribution, see for instance Dodson (1971) and Sampson (2009). Contrary to what is generally done in the literature, a key feature of this work is that fibres are modelled as two dimensional elements with an anisotropic hygro-elastic constitutive response. This allows to properly describe the role of the bonding regions in which the coupling between the hygroscopic and mechanical properties of fibres strongly influences the effective hygro-mechanical response (Bosco et al., 2015a; 2015b). The minimum micro-scale cell size, relative to the fibre length, necessary for the convergence of the effective properties is identified by analysing several network realizations. The resulting effective hygro-mechanical properties are investigated by studying the effects of different micro-structural characteristics, in particular the orientation distribution and the coverage, i.e. the ratio between the total area of the fibres and the cell area. Moreover, the adopted asymptotic homogenization approach enables to reconstruct all micro-structural fields, providing insight in the local deformation and displacement due to the interaction between hygroscopic and mechanical phenomena. Note finally that in the adopted homogenization formulation a small strain and geometrically linear regime has been assumed, at both the scales. This assumption is reasonable for (relatively) dense networks. For sparse networks large deformations at the network level, such as fibre buckling, may occur. These can be included by extending the homogenization procedure to a finite deformations formulation, along the lines of e.g. Temizer (2012). This is omitted here, but it may be considered as a topic for future research.

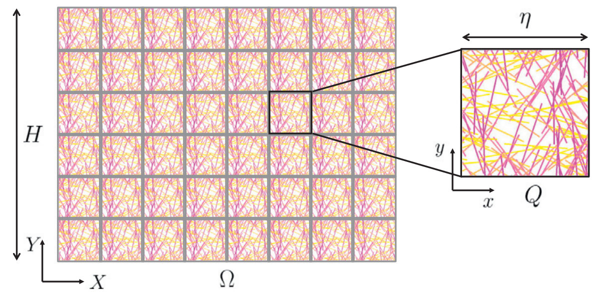

Fig. 1. Macroscopic domain and underlying periodic micro-structure.

This paper is organized as follows. In Section 2, a review on the asymptotic homogenization method is presented, focussing on the calculation of the effective hygro-thermo-mechanical material properties. The network generation and its constitutive characterization are introduced in Section 3. The adopted computational strategy is outlined in Section 4. The results of the study are illustrated in Section 5, in terms of the effective hygro-thermo-mechanical properties as well as the corresponding micro-structural fields. Conclusions are finally given in Section 6.

The following notations for Cartesian tensors and tensor products are used: a, $\mathbf{a}$, $\mathbf{A}$, and ${}^n\mathbf{A}$ denote, respectively, a scalar, a vector, a second-order tensor, and an nth-order tensor. Vector and tensor operations are defined as follows (employing Einstein's summation convention): the dyadic product $\mathbf{ab} = a_i b_j \mathbf{e}_i \mathbf{e}_j$, and the inner products $\mathbf{A}\cdot\mathbf{b} = A_{ij} b_j \mathbf{e}_i$, $\mathbf{A}\cdot\mathbf{B} = A_{ij} B_{jk} \mathbf{e}_i \mathbf{e}_k$, $\mathbf{A}:\mathbf{B} = A_{ij} B_{ji}$, with $\mathbf{e}_i$, $(i = x, y, z)$ the unit vectors of a Cartesian vector basis. Tensors and tensor operations are represented in a matrix form through Voigt notation: a column and a matrix of scalars are indicated by $\underset{\sim}{a}$ and $\underline{A}$, respectively. The matrix multiplication is defined as $(\underline{A}\underset{\sim}{b})_i = A_{ij} b_j$. Symbol ∇ indicates the gradient operator: $\nabla f = \partial f/\partial x\ \mathbf{e}_x + \partial f/\partial y\ \mathbf{e}_y + \partial f/\partial z\ \mathbf{e}_z$.

2. Asymptotic homogenization method

Consider a two dimensional domain $\Omega \in \mathbb{R}^2$, composed by the periodic repetition of a heterogeneous unit-cell of volume $Q \in \mathbb{R}^2$, as shown in Fig. 1. The following principles are formulated with reference to Sanchez Palencia (1980), Bakhvalov and Panasenko (1989) and Guedes and Kikuchi (1990). Denoting with H and η the characteristic sizes of the macroscopic domain and of the micro-structural unit-cell, respectively, it is assumed that a strong separation between scales exists, i.e. $\eta \ll H$. The field quantities governing the elastic problem (e.g. displacement, strain, stress) can thus be considered to vary smoothly at the macroscopic scale, whereas they are periodic at the micro-scale. For this reason, it can be assumed that all quantities explicitly depend on two variables: a slow (macroscopic) variable $\mathbf{X}$ and a fast (microscopic) variable $\mathbf{x} = \mathbf{X}/\eta$.

In the presence of macroscopic mechanical loads and moisture variations, and in the absence of body forces, the equilibrium equation reads

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{0} \tag{1}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, related to the gradient of the displacement field $\mathbf{u}$ through the constitutive relation

$$\boldsymbol{\sigma}(\mathbf{X}) = {}^4\mathbf{C}(\mathbf{x}) : (\nabla \mathbf{u} - \boldsymbol{\beta}(\mathbf{x})\chi(\mathbf{X})) \tag{2}$$

with ${}^4\mathbf{C}(\mathbf{x})$ the fourth order elasticity tensor and $\boldsymbol{\beta}(\mathbf{x})$ the second order hygro-expansion tensor, related to the moisture variation χ. They are periodic functions in the fast variable $\mathbf{x}$. Note that the thermo-mechanical problem obeys essentially the same equations,