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# Sensitivity of the mode locking phenomenon to geometric imperfections during wrinkling of supported thin films

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Although geometric imperfections have a detrimental effect on buckling, imperfection sensitivity has not been well studied in the past during design of sinusoidal micro and nano-scale structures via wrinkling of supported thin films. This is likely because one is more interested in predicting the shape/size of the resultant patterns than the buckling bifurcation onset strain during fabrication of such wrinkled structures. Herein, I have demonstrated that even modest geometric imperfections alter the final wrinkled mode shapes via the mode locking phenomenon wherein the imperfection mode grows in exclusion to the natural mode of the system. To study the effect of imperfections on mode locking, I have (i) developed a finite element mesh perturbation scheme to generate arbitrary geometric imperfections in the system and (ii) performed a parametric study via finite element methods to link the amplitude and period of the sinusoidal imperfections to the observed wrinkle mode shape and size. Based on this, a non-dimensional geometric parameter has been identified that characterizes the effect of imperfection on the mode locking phenomenon – the equivalent imperfection size. An upper limit for this equivalent imperfection size has been identified via a combination of analytical and finite element modeling. During compression of supported thin films, the system gets "locked" into the imperfection mode if its equivalent imperfection size is above this critical limit. For the polydimethylsiloxane/glass bilayer with a wrinkle period of 2 µm, this mode lock-in limit corresponds to an imperfection amplitude of 32 nm for an imperfection period of 5 µm and 8 nm for an imperfection period of 0.8 µm. Interestingly, when the non-dimensional critical imperfection size is scaled by the bifurcation onset strain, the scaled critical size depends solely on the ratio of the imperfection to natural periods. Thus, the computational data generated here can be generalized beyond the specific natural periods and bilayer systems studied to enable deterministic design of a variety of wrinkled micro and nano-scale structures.



## 1. Introduction

Recently, wrinkling of supported thin films has attracted significant attention both as a practical low-cost technique to fabricate micro and nano-scale patterns (Bowden et al., 1998; Kim et al., 2013; Li et al., 2013; Da et al., 2015; Stenberg et al., 2015; Saha and Culpepper, 2016) and as a model system to investigate the mechanics of complex pattern formation during self-organization (Huck et al., 2000; Groenewold, 2001; Brau et al., 2011; Cai et al., 2011; Breid and Crosby, 2011; Li et al., 2011; Cao and Hutchinson, 2012a). Wrinkled patterns are formed via compression of supported thin films and the mechanism is similar to Euler buckling of columns under compressive loads. Due to its ease of implementation, wrinkling has been extensively used in the past for applications such as tunable diffraction gratings (Kim et al., 2013; Da et al., 2015; Yu et al., 2010), patterned surfaces (Li et al., 2013; Stenberg et al., 2015; Kim et al., 2012; Sengupta Ghatak and Ghatak, 2013; Yang et al., 2013), micro and nano-fluidics (Huh et al., 2007; Chung et al., 2008), and nano-scale metrology (Stafford et al., 2005; Chung et al., 2011). As such, wrinkling of thin films provides a low-cost and scalable alternative to traditional capital-intensive microfabrication techniques for manufacturing of periodic micro and nano-scale patterns (Genzer and Groenewold, 2006; Mei et al., 2010; Ohzono and Monobe, 2012; Saha, 2014).

Although several analytical models have been developed to predict pattern formation during wrinkling (Groenewold, 2001; Brau et al., 2011; Cai et al., 2011; Jiang et al., 2007), they are inadequate for predicting complex wrinkled patterns that form due to geometric and/or material non-uniformity. Consequently, Finite Element Analysis (FEA) is an important tool for predictive design of wrinkled patterns (Saha and Culpepper, 2016; Huck et al., 2000; Cao and Hutchinson, 2012a, 2012b, Yin et al., 2014). Additionally,

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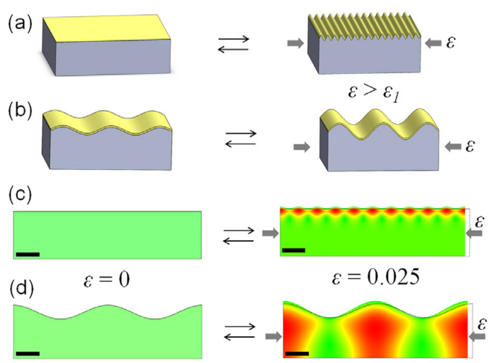


**Fig. 1.** Demonstration of the mode locking phenomenon. Schematic of formation of (a) natural wrinkles with small geometric imperfections and (b) mode locked wrinkles with large geometric imperfections. Finite element modeling of formation of (c) natural wrinkles with an imperfection amplitude of 2.4 nm and (d) mode locked wrinkles with an imperfection amplitude of 746 nm. Color information represents increasing first principal strain (from green to red). Scale bars are 2.5 µm long in (c) and (d). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

FEA enables one to probe the system and observe system parameters, such as deformation energy, that are not directly observable during physical experiments (Saha and Culpepper, 2016). Thus, FEA plays a critical role in both (i) predictive design and fabrication of engineered wrinkled patterns and (ii) analysis of the pattern formation behavior during wrinkling.

As wrinkled patterns are formed via buckling-based bifurcation, one must perturb the geometry or loading conditions to perform post-bifurcation analyses via finite element methods. Knowledge about the sensitivity of the pattern formation behavior to these perturbation-based imperfections is essential for an accurate analysis. For example, it is observed during FEA that no bifurcation occurs when a "small" imperfection is applied; whereas, the system transforms into a different one when a "large" imperfection is applied. Thus, one must carefully select the imperfection to accurately model the desired wrinkles.

Traditionally, selection of the appropriate geometric imperfections for wrinkling FEA has been performed by a trial-and-error technique wherein: (i) the shape is pre-selected to be the mode shape obtained from the pre-buckling modal analysis and (ii) the size is selected by identifying a small imperfection that still leads to wrinkles (Huck et al., 2000; Cao and Hutchinson, 2012b). Although effective, this technique is computationally inefficient because of the need to run modal analyses and several trial simulations. A more efficient technique would be to select a kinematically admissible mode shape with a size that is large but lower than the limit beyond which the system behaves differently. In addition to the computational advantage, identifying this upper limit has practical import for design of engineered wrinkles both when this change in behavior is undesirable for high-quality patterning or desirable for generating complex shapes. Recently, we have empirically demonstrated that when a large imperfection is applied to a supported thin film, it is possible to fabricate complex hierarchical wrinkles by first "locking" the system into an otherwise energetically unfavorable mode (Saha and Culpepper, 2016). In such systems, the imperfection mode grows with increasing compressive strain in exclusion to the energetically favorable natural mode for the corresponding "perfect" system due to the phenomenon of mode lock-in. This mode locking phenomenon is illustrated for the case of a large imperfection size in Fig. 1. Later in Section 5, it has been demonstrated that such mode locking behavior is observed even for moderately large imperfections. In this article, I have (i) identified the characteristics of this mode locking phenomenon and (ii) quantified the sensitivity of this mode locking behavior to geometric imperfections in the system.

The sensitivity of the mode lock-in phenomenon to geometric imperfections has been studied during bilayer wrinkling by systematically perturbing the mesh with periodic sinusoidal imperfections. As commercial FEA software packages lack the tools to introduce well-controlled arbitrary mesh perturbations, a custom mesh perturbation toolbox (Saha and Culpepper, 2014) using the commercial MATLAB package has been developed and validated. This toolbox has been interfaced with the commercially available COMSOL 5.1 FEA package to perform post-buckling analyses of wrinkled patterns. Based on these analyses, I have (i) quantified the minimum size of imperfections for onset of mode lock-in and (ii) linked the minimum imperfection size to the bilayer properties. These computational results enable one to identify process control limits during fabrication of high-quality wrinkles. In addition, characterization of the mode locking behavior opens the door to explore the role of mode locking in designing and fabricating such complex wrinkled patterns in "imperfect systems" that are otherwise energetically unfavorable and are physically unobservable in the corresponding "perfect systems".

## 2. Background

### 2.1. Wrinkling process physics

Wrinkling of compressed bilayers is an affordable fabrication technique for generating a variety of complex periodic micro and nano-scale patterns. The morphology of the wrinkles depends on the geometry, material properties, and the strain (Groenewold, 2001; Brau et al., 2011; Cai et al., 2011; Breid and Crosby, 2011; Chiche et al., 2008). Herein, I have investigated the mode locking behavior within the context of the simplest wrinkle pattern – 1-D periodic sinusoidal wrinkles that are generated via uniaxial compression of a bilayer. A schematic of the bilayer wrinkling process is illustrated in Fig. 1(a). Essential elements of these bilayer systems are: (i) a film that is *thin* relative to the base, (ii) mismatch in the elastic moduli of the film and the base with the film being stiffer than the base, and (iii) loading conditions that generate in-plane compressive strain in the film. In such systems, the state of pure compression becomes unstable beyond a critical strain and wrinkles are formed via periodic bending of the film.

The period and amplitude of these patterns can be tuned by controlling the (i) thickness of the top layer, (ii) compressive strain in the top layer, and (iii) ratio of stiffness moduli of the top and the bottom layers. The period is determined by the sinusoidal mode shape that minimizes the deformation energy of the film and the base; whereas, the amplitude is determined by kinematics. At the onset of wrinkling, the natural period ($\lambda_n$) and amplitude ($A$) of the wrinkles can be estimated in terms of the top stiff layer thickness ($h$) as (Groenewold, 2001; Chiche et al., 2008):

$$\lambda_n = ch\eta^{1/3} \tag{1}$$

$$A = \frac{\lambda_n}{\pi}(\varepsilon - \varepsilon_1)^{1/2} \tag{2}$$

Here, '$c$' is a non-dimensional parameter that depends on the Poisson's ratio of the materials, $\eta$ is the ratio of Young's moduli of the film to the base, $\varepsilon$ is the applied compressive strain, and $\varepsilon_1$ is the first bifurcation strain above which wrinkled patterns are observed. This bifurcation strain is determined entirely by the material properties and is given by (Groenewold, 2001; Chiche et al., 2008):

$$\varepsilon_1 = \pi^2 c^{-2}\eta^{-2/3} \tag{3}$$