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Analytical study of electromechanical buckling of a micro spherical elastic film on a compliant substrate part I: Formulation and linear buckling of periodic patterns

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ABSTRACT

This work presents an analytical study of the electromechanical buckling of a micro spherical thin film bonded to a compliant elastic substrate. The spherical film is subjected to electrostatic attraction forces that are induced by applying a voltage difference between an outer spherical elastic film electrode and inner rigid, fixed and grounded spherical electrode. When the applied voltage is small, the film contracts while maintaining its complete spherical shape. However, when the applied voltage reaches a critical value, the film buckles into high-ordered periodic patterns (i.e. elastic surface wrinklings). Motivated by experimental results of other studies, this work examines the critical buckling state of one-dimensional, square checkerboard and hexagonal patterns. As will be shown, the above considered patterns are associated with the same critical state, and therefore, all patterns have equal buckling voltage and critical wavelength. Furthermore, with increasing radius of the film/substrate system the electromechanical buckling response converges to the electromechanical pull-in instability of the well-known two parallel plate electrodes, as will be revealed by the asymptotic analytical solution. Finally, the elastic ripples of the electromechanically buckled film can be generated or removed by a simple On/Off switching of the applied voltage. The ability to generate and remove elastic ripples tremendously increases the potential of such microsystem to be utilized in different applications in the field of Micro and Nano electromechanical systems.

1. Introduction

Over the past decade, researchers have devoted ongoing attention to the buckling of stiff thin films bonded to an elastic compliant substrate. The pioneering work of Bowden et al. (1998) showed that such a buckling response generates well-defined periodic pattern structures characterized by wavelength dimensions ranging from 0.1 to 100 µm. The high ordered periodic buckling patterns of the films have numerous potential applications, such as biocompatible topographic matrices for cell alignment (Song et al., 2008b), biomimetic fingerprints and identity tags (Bae et al., 2015; Yin and Boyce, 2015), optical grating, optical On/Off switchers (Elata and Abu-Salih, 2006) and self-assembly of materials (Cao et al., 2008; Chen, 2013). The determination of the elastic surface wrinklings of a buckled spherical thin film on a compliant substrate was studied by Stoop et al. (2015) and Li et al. (2011). The theoretical analysis in those studies start with the formulation the total strain energy of the film/substrate system and then apply the variational calculus method for deriving the equilibrium equation. The buckling of the film was induced by applying pressure difference on the film/substrate system or by swelling and deswelling of the soft gel substrate. The influence of the film's curvature on the selection of the elastic surface patterns has been addressed. As reported, the elastic surface pattern transform from hexagonal to labyrinth pattern with decreasing the curvature of the film. The elastic buckling of a spheroidal thin film/substrate system with application to morphogenesis and morphologies of biological systems has been studied (Chen and Yin, 2010; Yin et al., 2009). The periodic pattern of buckled thin film layers embedded in a soft medium has been studied by Rudykh and Boyce (2014), Li et al. (2013) and Slesarenko and Rudykh (2016). In their works, it has been reported that periodic patterns of the postbuckling state of the interfacial layers can be utilized for tuning elastic wave propagation in deformed materials (Galich and Rudykh, 2016).

Most of those studies investigate the buckling of metal thin film on a compliant planar substrate. In this case, buckling may take place either by heating the film/substrate system or by applying compression pre-stress or pre-strain. In the compression approach,

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the pre-strains can be obtained by depositing the film layer on an initially tensioned substrate. In this case, the magnitude of the pre-strain is determined by the magnitude of applied pre-tension. In the heating approach, buckling occurs due to the thermal mismatch (i.e. different thermal expansion coefficients) between the film and the substrate (Abu-Salih and Elata, 2005; Bowden et al., 1998; Cai et al., 2011; Song et al., 2008b).

The critical buckling state of a thin film bonded to an elastic compliant substrate is generally solved using the corresponding Föppl-von-Kármán plate equilibrium equations (Audoly and Boudaoud, 2008a; Cai et al., 2011), while postbuckling analysis, in case of periodic patterns, is generally based on the total strain energy method (Abu-Salih, 2016b; Audoly and Boudaoud, 2008b; Chen and Hutchinson, 2004). Recent studies investigated the buckling response through the total strain energy analysis (relative to the unbuckled state) of high-ordered: one-dimensional, square-checkerboard, hexagonal and herringbone periodic patterns (Cai et al., 2011; Li et al., 2011; Song et al., 2008a). The study of periodic patterns of buckled film was motivated by experimental observations (Bowden et al., 1998; Cai et al., 2011; Cao et al., 2008; Song et al., 2008b). The critical buckling strain (or stress) and the secondary bifurcation instabilities of the film are derived from the corresponding analysis of the total strain energy of each postulated periodic patterns.

The labyrinth and triangle patterns of a buckled micro spherical SiO_2 thin film bonded to a spherical Ag core was studied and experimentally verified by Cao et al. (2008). Labyrinth and triangle patterns emanate from pattern transition of herringbone and hexagonal patterns, respectively. As reported in their study, the hexagonal pattern is more stable and thus is preferred at large curvatures of the film, while the herringbone pattern dominates when the curvature is small. In addition, the transition from a hexagonal pattern to a herringbone pattern (i.e., secondary bifurcation instability) is achieved either by increasing the stresses or by decreasing the curvatures of the spherical film.

The development of micro fabrication processes in the fields of Micro and Nano Electro-Mechanical-Systems (MEMS and NEMS, respectively) in the last decade has made it possible to fabricate micro elastic structures such as beams, plates and thin shells. In addition, the development of metal deposition has facilitated fabricating micro and nano thin metal layers on a different polymer substrate materials, such as PDMS, SU-8 and polyimide substrates (Senturia, 2004). Micro thin shells, beams and plates are implemented in different micro electromechanical sensors and actuators. Such micro elastic structures can be actuated by electrostatic, thermal or mechanical loads. The electromechanical buckling response of a micro elastic beam that is simultaneously subjected to electrostatic attraction force and a pre-stress has been studied and experimentally validated. As reported, mechanical buckling of a micro beam can be instigated by electrostatic forces. The electromechanical buckling of micro beam and of thin film bonded to an elastic foundation, has the potential to be implemented in different MEMS applications, such as measuring residual stresses and On/Off electrostatic optical switches (Abu-Salih, 2014; Abu-Salih and Elata, 2006).

In all previous studies the elastic surface wrinkling patterns are activated by either applying internal pressure, in-plane compressive pre-stress (or pre-strain), thermal mismatch, or swelling and deswelling (i.e. shrinking) of the substrate. These activating mechanisms (or methods) are may complicate the design of a microsystem and are not well controllable in MEMS technology. The most disadvantage of these previous methods stems from the issue of needing additional micro components, such as micro valves and micro resistor in order to be employed in MEMS devices. In contrast, the activation of surface wrinkling patterns by means of electrostatic forces can be triggered by simple On/Off switching of the applied voltage. The electrostatic actuation mechanism can be easily integrated in micro systems, and therefore, it may be implemented in different applications such as deformable micro mirrors, optical switches, and in adaptive aerodynamic drag control systems. The activation of elastic ripples of the spherical film surface by applying electrostatic forces has not been studied before. Furthermore, some question such as how the curvature of the spherical film and the initial gap between the two spherical electrodes affects the selection of the wrinkling pattern, needs to be addressed.

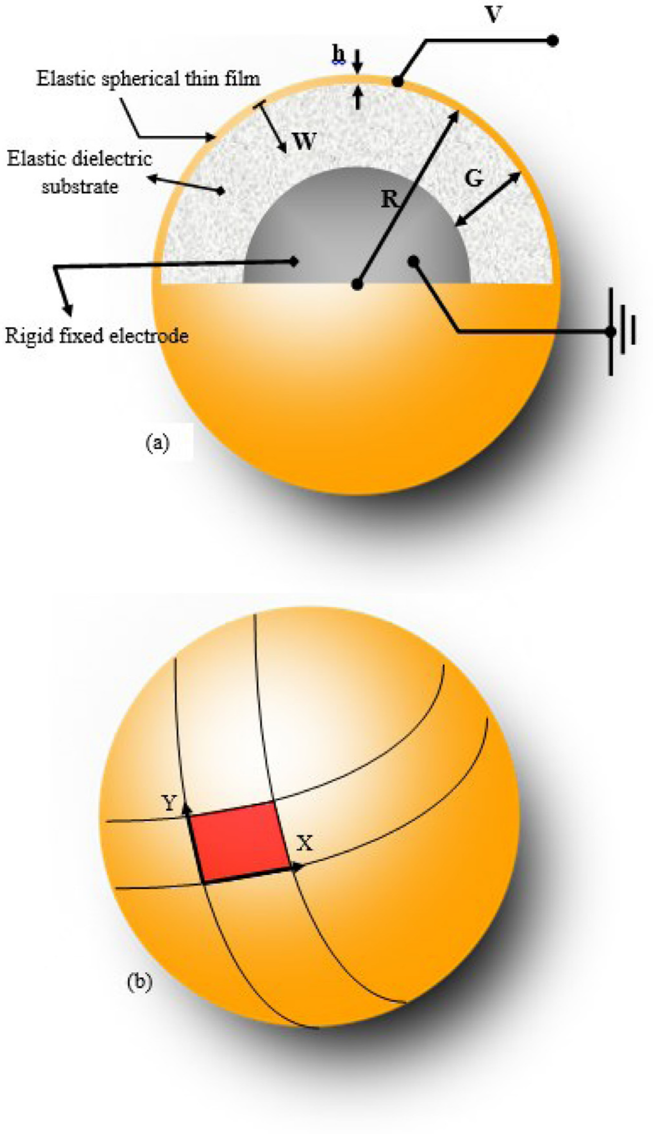

Fig. 1. (a) Schematic view of an elastic spherical thin film bonded to a spherical elastic dielectric substrate. The dielectric substrate is bonded to an inner fixed and rigid spherical grounded electrode. The voltage difference *V* is applied on the film electrode. (b) Schematic view of one cell of the film with its corresponding local coordinate system *X* and *Y*.

This work presents an analytical study of the electromechanical buckling of an elastic spherical film electrode bonded to an elastic compliant substrate. The electromechanical system consists of an outer elastic thin film electrode that is bonded to a dielectric substrate. The elastic dielectric substrate is bonded to an inner rigid and electrically grounded electrode, as schematically illustrated in Fig. 1. The electromechanical buckling response is induced by applying voltage difference between the elastic film electrode and the