International Journal of Solids and Structures 109 (2017) 199–209

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Extended displacement discontinuity method for an interface crack in a three-dimensional transversely isotropic piezothermoelastic bi-material. Part 2: Numerical method

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ARTICLE INFO

Article history:
Received 9 October 2016
Revised 16 December 2016
Available online 16 January 2017

Keywords:
Three-dimensional transversely isotropic piezothermoelastic bi-material
Interface crack
Extended displacement discontinuity method
Triangular element
Fundamental solutions
Gaussian distribution function
Electric and thermal boundary conditions
Extended stress intensity factors
Energy release rate (ERR)
Local *J*-integral

ABSTRACT

The extended displacement discontinuity method is proposed to analyze the nonlinear electric and thermal effects on an interface crack in a three dimensional (3D), transversely isotropic, piezothermoelastic bi-material under combined mechanical-thermo-electrical loadings. The fundamental solutions for uniformly distributed, extended displacement discontinuities applied over a triangular element are obtained by integrating the fundamental solutions for the unit-point, extended displacement discontinuities given by Part 1 over the triangular area. In order to eliminate the oscillatory singularity of the stresses near the crack front, the Delta function in the fundamental solutions is approximated by the Gaussian distribution function, and accordingly, the Heaviside step function is replaced by the Error function. The extended, displacement discontinuity boundary element method with an iterative approach is proposed to determine the value of the fields in the crack interior for opening-crack model. As an example, an elliptical interface crack is studied under different electrical and thermal boundary conditions. The extended stress intensity factors (SIFs) without oscillatory singularities, the forms of the energy release rate (ERR) and local *J*-integral which can be both expressed in terms of intensity factors are obtained. The numerical method is validated and the influence of combined loadings and material-mismatch on the results is studied. The effects of different boundary conditions and ellipticity ratio on the results are also investigated.

1. Introduction

In Part 1 (Zhao et al., 2016), the analytical solution for an arbitrarily shaped interface crack in a 3D piezothermoelastic bi-material subjected to combined mechanical-electro-thermal loadings is obtained. The fundamental solutions for the extended displacements and stresses under unit-point extended displacement discontinuities are derived, and the corresponding boundary integral-differential equations are obtained. The extended stress intensity factors (SIFs) and local *J*-integral as well as energy release rate (ERR) are all derived in terms of the extended displacement discontinuities across the interface crack faces. As the anlaytical solution is quite limited and can only be obtained for a certain shaped crack under uniformly distributed loadings rather than an arbitrarily shaped crack under complex combined loadings, the numerical method is thus in need for more complicated cases.

Among various numerical methods, the displacement discontinuity method (DDM) was first proposed by Crouch (1976) to solve crack problems, and it was found flexible and efficient in studying crack problems. This method was then extended to analyze piezoelectric media (Zhao et al., 1997; Fan et al., 2009) and magneto-electro-elastic material (Zhao et al., 2007, 2015a, b) by extending the conventional elastic displacement discontinuity to electric and magnetic potential discontinuities.

It is observed from Part 1 that there exists oscillatory singularity in the interface crack problems, as is similar to the piezoelectric bi-materials (Zhao et al., 2004), and the arisen oscillatory singularity will induce the overlapping of the cracks faces. However, this phenomenon is unreasonable and unrealistic, thus many researches were conducted to remove the oscillatory singularity. For instance, Hermann and Loboda (2003a, b) utilized the contact-zone model of interface crack, Zhang and Wang (2013) and Zhao et al. (2014b, 2015a,b) approximated the Delta function with Gaussian distribution function to eliminate the oscillatory singularity.

Based on the obtained fundamental solutions for a unit-point extended displacement discontinuity given in Part 1, the fun-

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http://dx.doi.org/10.1016/j.ijsolstr.2017.01.024
0020-7683/

damental solutions for a triangular element are derived in this paper. The Delta function in the integral-differential equations is approximated and replaced by the Gaussian distribution function as in Zhang and Wang (2013a) and Zhao et al., (2014b, 2015a, b) to remove the oscillatory singularity.

The boundary condition is a crucial factor that affects the fracture behavior of the interface crack in piezothermoelastic bi-materials, and it becomes more complicated compared with piezoelectric materials. Gao and Wang (2001) solved a permeable N-collinear crack in a piezothermoelastic material subjected to uniformly applied mechanical-electric loading associated with uniform heat flux at infinity. Zhong and Zhang (2013) proposed an opening crack model for piezothermoelastic materials that took into consideration the effects of applied mechanical-electric loadings on the thermal stress filed near the crack tip, where the electric displacement and heat flux on the crack faces were assumed to be related with the crack opening displacement. Besides, Zhang and Wang (2013b) discussed the application and effects of the five electric and thermal boundary conditions on the crack faces on the fracture behavior of piezothermoelastic materials in detail. Later Zhang and Wang (2015) took the Maxwell stresses into consideration to investigate the crack problem in piezoelectric material under combined mechanical-electric-thermal loadings.

Making use of the obtained fundamental solutions and taking boundary conditions into consideration, the boundary extended displacement discontinuity method is proposed here to analyze an elliptical interface crack in a 3D, transversely, isotropic, piezothermoelastic bi-material as an example to check the proposed numerical method and the analytical solution. The effects of different electric and thermal boundary conditions and the material-match as well as the ellipticity ratio on the fracture behavior of the crack are demonstrated, and some interesting phenomena are spotted.

This paper is organized as follows: the statement of the crack problem and the detailed five boundary conditions are presented in Section 2, and the extended displacement discontinuity boundary integral-differential equations are listed in Section 3. The extended displacement discontinuities over a triangular element are derived and the oscillatory singularity is removed by approximating the Delta function with Gaussian distribution function in Section 4. In Section 5, the extended stress intensity factors (SIFs), the local J-integral as well as the ERR are derived in terms of the extended displacement discontinuities. In Section 6, the numerical method with an iterative method to determine the electric and thermal fields in the crack interior is proposed to study an elliptical interface crack as an example. In Section 7, the correctness of the method is validated, and the effects of the combined mechanical-thermal-electric loadings and different boundary conditions as well as the material-mismatch and ellipticity ratio on the fracture behavior are all investigated. The concluding remarks are drawn in Section 8.

2. Statement of the problem

Consider a 3D, two-phase, transversely isotropic, piezothermoelastic medium with the interface parallel to the plane of isotropy. A Cartesian coordinate system is set up with the xoy plane coinciding with the interface, and the polarization direction is along the z-direction. The two perfectly bonded dissimilar solids are assumed to occupy the upper and lower half-spaces, denoted as material 1 and 2, respectively. There exists an arbitrarily-shaped interface crack S lying at the interface plane xoy, and the upper and lower surfaces of crack S are denoted by S^+ and S^-, respectively. The distributed combined loadings, including mechanical loading p_z, steady heat flux h_z, and electric displacement component D_z, are applied, as schematically shown in Fig. 1. Due to the perfect adhesion between the solids outside the crack along

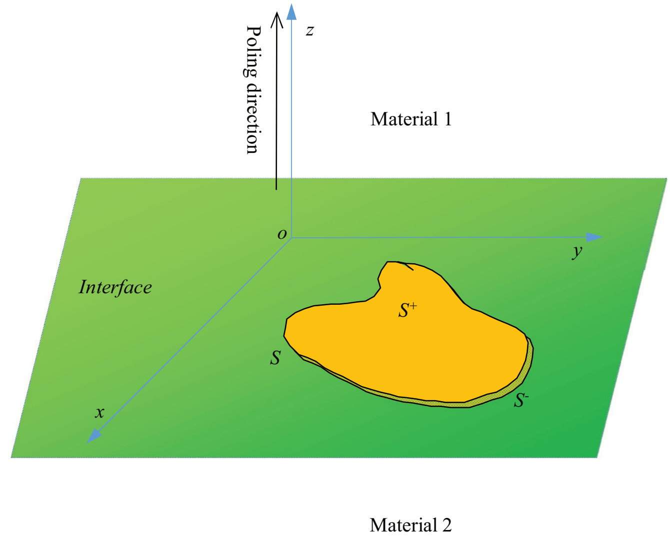

Fig. 1. An arbitrarily shaped interface crack S lying in the interface plane perpendicular to the poling direction.

the common interface, the extended stress and displacement components are continuous along the interface outside the crack S. The outer normal vectors of S^+ and S^- have the relation

$$\{n_i\}^+ = \{0, 0, -1\}, \quad \{n_i\}^- = \{0, 0, 1\}. \tag{1}$$

When applied with combined mechanical-thermo-electro loadings, the interface crack may be opened and filled by a medium with a certain thermal conductivity and dielectric permittivity. Consequently, the electric and thermal boundary conditions on the interface crack faces can be quite complicated. There are totally five types of boundary conditions to take into account (Zhang and Wang, 2013), which are listed as follows with D_z^c and h_z^c denoting the electric displacement and heat flux in the z-axis direction in the crack interior:

$$\text{case1}: \quad \begin{array}{l} D_z^c(x,y,0^+) = D_z^c(x,y,0^-) = 0, \\ h_z^c(x,y,0^+) = h_z^c(x,y,0^-) = 0, \end{array} \tag{2a}$$

for an electrically and thermally impermeable crack;

$$\text{case 2}: \quad \begin{array}{ll} \|\varphi\| = 0, & D_z^c(x,y,0^+) = D_z^c(x,y,0^-), \\ \|\theta\| = 0, & h_z^c(x,y,0^+) = h_z^c(x,y,0^-), \end{array} \tag{2b}$$

for an electrically and thermally permeable crack;

$$\text{case 3}: \quad \begin{array}{l} D_z^c(x,y,0^+) = D_z^c(x,y,0^-) = 0, \\ \|\theta\| = 0, \quad h_z^c(x,y,0^+) = h_z^c(x,y,0^-), \end{array} \tag{2c}$$

for an electrically impermeable and thermally permeable crack;

$$\text{case 4}: \quad \begin{array}{l} \|\varphi\| = 0, \quad D_z^c(x,y,0^+) = D_z^c(x,y,0^-), \\ h_z^c(x,y,0^+) = h_z^c(x,y,0^-) = 0, \end{array} \tag{2d}$$

for an electrically permeable and thermally impermeable crack;

$$\text{case 5}: \quad \begin{array}{l} D_z^c(x,y,0^+) = D_z^c(x,y,0^-) = -\kappa^c\|\varphi\|/\|w\|, \\ h_z^c(x,y,0^+) = h_z^c(x,y,0^-) = -\beta^c\|\theta\|/\|w\|, \end{array} \tag{2e}$$

for an electrically and thermally semi-permeable crack (Zhang and Wang, 2013b), or opening crack model (Zhong and Zhang, 2013). Here ε^c and β^c denote the dielectric and heat conduction in the crack interior, respectively.

When dealing with a crack in an infinite media under far-field loadings, one can transfer the far-field loadings onto the crack faces. Thus the crack problem can be treated as the superposition of crack-free problem and perturbed problem. In order to satisfy the traction-free condition on the crack faces in the original problem, the loadings applied on the crack faces in the