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Size-dependent effective behaviors of multiferroic fibrous composites with interface stress

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ABSTRACT

The objective of this work is to study the size-dependent effective behaviors of multiferroic fibrous composites with interface stress subjected to generalized in-plane deformation and out-of-plane electromagnetic fields. Using the complex variable approach together with several micromechanical models, closed-form expressions of the effective moduli are obtained. In contrast to the composite with perfect bonding, the effective property formulations show the dependence on the size of the fibers. The derived solutions are applied to several examples to demonstrate the effect of interface stress, and to examine the results among different micromechanical models. Numerical results show that while most of the effective moduli increase as the radius of the fiber decreases, the magnetoelectric voltage coefficient, the figure of merit of the multiferroic material, decreases. Further, when considering the effective transverse shear modulus of composite with interface stress, a higher order approximation is required for the equivalency between an interphase and an imperfect interface if the interphase is not sufficiently thin or is very stiff.

1. Introduction

The magnetoelectric (ME) coupling in multiferroic composite is of interest for a variety of applications ranging from the magnetic field detectors to four-state memory cells (Fiebig, 2005; Nan et al., 2008). The ME effect in the multiferroic composite is achieved by the mechanical interaction between the piezoelectric and magnetostrictive/piezomagnetic phases. Therefore, the interface condition plays an important role in achieving the giant magnetoelectricity. The majority of studies concerning the multiferroic composites with imperfect interface were primarily concerned with the spring-type imperfection at which a thin, soft and weakly conducting interphase occurs between the constituent phases (Bichurin et al., 2003; Nan et al., 2003; Chang and Carman, 2007; Wang and Pan, 2007; Kuo and Huang, 2016). Exceptions are Pan et al. (2009), Kuo (2013), and Kuo and Chen (2015) who investigated the *interface stress* effect on the multiferroic fibrous composite. However, all of the mentioned studies on the interface effect are limited to the fibrous composites subjected to the generalized *anti-plane* deformation due to the complexity.

The surface or interface stress in composites has been the focus of research in recent years. This effect is particularly important in composites containing small size particles, since the smaller the particle size, the larger will be the total contact interface between the particles and matrix. The study of the surface/interface stress effect in solids can be traced back to 1975 when Gurtin and Murdoch (1975) using the membrane theory to propose a theoretical analysis for an interface between two different solids with interface stress. This interface relation is a generalization of the Young–Laplace equation where the surface stress arises and is proportional to the surface strain.

In the literature, two kinds of model are widely used to simulate the composite with interface stress, namely, the *interface stress* model and the *interphase* model. In elasticity, the interface stress model characterizes the imperfection by a jump in the traction across the interface but yet maintaining the continuity of the displacement field. The interphase model, on the other hand, is a three-phase configuration which models a thin and stiff interphase between the particle and matrix phases. Perfect bonding is assumed to prevail at both the particle/interphase and interphase/matrix interfaces. Specifically, the interface stress model can be considered as the limiting case of the interphase model by making an asymptotic analysis for the stiffer interphase of small uniform thickness (Benveniste and Miloh, 2001).

Relevant studies on the composites with interface stress have been investigated in various physics phenomenon and overall behaviors of nano-composites. For instance, in the context of elasticity, Chen et al. (2007a, 2007b) assessed the overall behavior of composites containing spherical or cylindrical inclusions with

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the surface effect. Mogilevskaya et al. (2008) proposed a semi-analytical method to solve the problem of multiple interacting circular nano-inhomogeneities, while Duan et al. (2005) and Kushch et al. (2011, 2013) took into account the interface stress to estimate the effective moduli of solids containing nano-spherical inhomogeneities. In the context of heat conduction, Torquato and Rintoul (1995) gave the rigorous bounds on the effective conductivity of random composites in terms of the phase contrast between the inclusion and matrix, the interface strength and higher-order morphological information. Cheng and Torquato (1997) studied the overall conductivity of periodic composites by Rayleigh's method for the case of highly conducting interface. Miloh and Benveniste (1999) investigated the effective conductivity of composite with ellipsoidal inhomogeneities and highly conducting interfaces. Related subjects on the highly conducting interface also include Lipton (1997), Le-Quang et al. (2010), and Kushch et al. (2015). Additional examples are encountered in the context of wave propagation by, for example, Olsson et al. (1990), Wang et al. (2007), and Kuo and Yu (2014). Furthermore, the microstructure-independent exact connections, the Levin's formula and Hill's connections, are generalized to the nano-composites containing spherical particles or cylindrical fibers with interface stress in the setting of thermoelasticity and piezoelectricity by Chen and Dvorak (2006), Duan and Karihaloo (2007), Chen (2008) and Benveniste (2014).

In a departure from previous works, in this paper we consider the effect of interface stress on the effective moduli of multiferroic fibrous composites subjected to the generalized *in-plane* deformation ($\varepsilon_{13}=0,\ \varepsilon_{23}=0,\ \varepsilon_{33}\neq 0$) coupled to out-of-plane electromagnetic fields. We begin in Section 2 by addressing the interface stress model for a multiferroic fibrous composite with transversely isotropic constituents. The generalized in-plane deformation with transverse electromagnetic fields are applied at the remote boundary of the composite. In Section 3, we establish the framework of the effective property of the composite while we derive the closed-form solutions of the macroscopic behaviors by different micromechanical models in Section 4. The micromechanical models used include the dilute approximation, Mori–Tanaka method, composite cylinder assemblage model/generalized self-consistent method, and interphase model. These methodologies are illustrated in Section 5 using composites made of $BaTiO_3$ and $CoFe_2O_4$. The effect of higher-order interface stress is also discussed to resolve the discrepancy of results predicted by the interphase model and other models.

2. Interface model

Let us consider a composite consisting of a matrix in which parallel and separated nano-fibers are embedded with interface stress. The fibers and the matrix are made of transversely isotropic magnetoelectroelastic materials, and the fibers are of circular cross sections with radius a. We choose a cylindrical coordinate system $(r,\ \theta,\ x_3)$ such that x_3-axis is parallel to aligned nano-fibers, and $(r,\ \theta)$ plane is perpendicular to them.

Assume that this two-phase multiferroic composite is under remote in-plane mechanical strains ε^0_{rr}, $\varepsilon^0_{\theta\theta}$ and $\varepsilon^0_{r\theta}$, and uniform strain ε^0_{33}, electric field E^0_3, and magnetic field H^0_3 along the x_3-direction. The constitutive laws of the constituents for generalized in-plane deformation with out-of-plane electromagnetic fields in cylindrical coordinates, therefore, can be characterized in a compact form as (Benveniste, 2014)

$$\mathbf{\Sigma}=\mathbf{L}\mathbf{Z}, \tag{2.1}$$

where

$$\mathbf{\Sigma}=\begin{pmatrix}\sigma_{rr}\\ \sigma_{\theta\theta}\\ \sigma_{33}\\ \sigma_{r\theta}\\ D_3\\ B_3\end{pmatrix},\quad \mathbf{L}=\begin{pmatrix}k+G & k-G & l & 0 & e_{31} & q_{31}\\ k-G & k+G & l & 0 & e_{31} & q_{31}\\ l & l & n & 0 & e_{33} & q_{33}\\ 0 & 0 & 0 & G & 0 & 0\\ e_{31} & e_{31} & e_{33} & 0 & -\kappa_{33} & -\lambda_{33}\\ q_{31} & q_{31} & q_{33} & 0 & -\lambda_{33} & -\mu_{33}\end{pmatrix},$$

$$\mathbf{Z}=\begin{pmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{33}\\ 2\varepsilon_{r\theta}\\ -E_3\\ -H_3\end{pmatrix}. \tag{2.2}$$

Here σ_{ij} and ε_{ij} stand for the stress and strain; D_3, E_3, B_3 and H_3 are electric displacement, electric field, magnetic flux, and magnetic field in the x_3-direction, respectively. The coefficients k, l, n, and G are the Hill's phase moduli in transverse isotropy; e_{ij}, q_{ij}, κ_{33}, μ_{33} and λ_{33} are the piezoelectric coefficient, piezomagnetic coefficient, dielectric permittivity, magnetic permeability, and ME coupling coefficient.

The strains, electric field and magnetic field can be derived from the gradient of elastic displacements u_r, u_θ, electric potential ϕ, and magnetic potential ψ as follows:

$$\varepsilon_{rr}=\frac{\partial u_r}{\partial r},\ \ \varepsilon_{\theta\theta}=\frac{1}{r}\left(u_r+\frac{\partial u_\theta}{\partial\theta}\right),\ \ 2\varepsilon_{r\theta}=\frac{1}{r}\frac{\partial u_r}{\partial\theta}+\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r},$$
$$\varepsilon_{33}=\frac{\partial u_3}{\partial x_3},\ \ E_3=-\frac{\partial\phi}{\partial x_3},\ \ H_3=-\frac{\partial\psi}{\partial x_3}. \tag{2.3}$$

Now consider that the imperfection interface is a mechanically stiff and highly electromagnetic conducting interface, which is a generalization of the membrane-type model. This model implies that the normal fluxes have a jump on the interface boundary ∂V_i, while the displacement and potentials are continuous (Benveniste and Miloh, 2001). Moreover, due to the nature of the remote loading and the cylindrical geometry one has $\frac{\partial}{\partial x_3}=0$, so that the interface conditions reduce to

$$[[\sigma_{rr}-i\sigma_{r\theta}]]_{\partial V_i}=\left.\frac{\sigma^s_{\theta\theta}}{r}\right|_{\partial V_i}+i\left.\frac{\partial\sigma^s_{\theta\theta}}{r\partial\theta}\right|_{\partial Vi},$$
$$[[u_r+iu_\theta]]_{\partial V_i}=\mathbf{0}. \tag{2.4}$$

Here a jump of a quantity η across ∂V_i is denoted by $[[\eta]]_{\partial Vi}=\eta^{(m)}-\eta^{(i)}$, $i=\sqrt{-1}$, and the indices m, i, s denote the quantity pertaining to the matrix, inclusion, and interface, respectively.

We assume that the interface is isotropic, and the material properties are distinct from those of the constituents. Thus, in a cylindrical coordinate system defined at the center of each fiber, the constitutive relations of the interface are given by (Sharma et al., 2003)

$$\begin{pmatrix}\sigma^s_{\theta\theta}\\ \sigma^s_{33}\\ D^s_3\\ B^s_3\end{pmatrix}=\begin{pmatrix}L^s_{\theta\theta} & L^s_{\theta z} & 0 & 0\\ L^s_{\theta z} & L^s_{\theta\theta} & 0 & 0\\ 0 & 0 & -\kappa^s_{33} & 0\\ 0 & 0 & 0 & -\mu^s_{33}\end{pmatrix}\begin{pmatrix}\varepsilon^s_{\theta\theta}\\ \varepsilon^s_{33}\\ E^s_3\\ H^s_3\end{pmatrix}. \tag{2.5}$$

3. Effective moduli

Consider a representative volume element (RVE) consisting of volume V of the multiferroic composite. Based on the homogenization method for the composite material, the effective magnetoelectroelastic parameter $\mathbf{L}^*$ of the composite is defined as

$$\langle\mathbf{\Sigma}\rangle=\mathbf{L}^*\langle\mathbf{Z}\rangle. \tag{3.1}$$

For the composite with mechanically stiff and highly electromagnetic conducting interfaces, the averages of the generalized stress