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A new stable inverse method for identification of the elastic constants of a three-dimensional generally anisotropic solid

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ABSTRACT

This article presents a new approach for inverse identification of all elastic constants of a 3D generally anisotropic solid with arbitrary geometry via measured strain data. To eradicate the nonlinear inequality constraints posed on the elastic constants, the problem is first transformed to an unconstrained one by the Cholesky factorization theorem. The cost function is defined by the Tikhonov regularization method, and the inverse problem is solved using the damped Gauss-Newton technique, where a meshless method is employed for the direct and sensitivity analyses. To demonstrate the effectiveness of the proposed approach, several examples are presented in the end, where all experimental data are numerically simulated. Analyses of these examples show that all twenty-one elastic constants of an example material can be correctly identified even when measurement errors are relatively large and initial guesses are far from exact values.

1. Introduction

In engineering industries, anisotropic materials have been extensively used for various applications. The fourth-order tensor of elasticity for a generally anisotropic elastic material is expressed in terms of 21 independent elastic constants (Sadd, 2009), each of which cannot be measured separately. For evaluation of these constants, it is advantageous to use displacements/strains as measured data, obtained from static experiments without specific limits on specimen size and geometry. Over the years, a great amount of identification works have been reported in this regard either for isotropic or anisotropic materials. Although the amount of research is extensive in this area, only a few of such works are reviewed herein, especially for anisotropic materials.

Wang and Kam (2000) presented an algorithm of multi-start global optimization, based on the finite element method (FEM) to identify the five elastic constants of laminated composite plates. By considering the lower/upper bounds of the elastic constants, they formulated this problem as a constrained minimization problem. Shin and Pande (2003) presented a finite element-based method to identify the nine elastic constants of an orthotropic material with known principal material axes. In their method, the displacements measured at several points were used to train a neural network model, by which the unknown material parameters were computed. Huang et al., (2004) presented an inverse technique based on the boundary element method (BEM) and the Levenberg-Marquardt method for identification of elastic parameters of 2D orthotropic bodies. They used displacement measurements to identify the four elastic constants of 2D orthotropic materials. For identification of the six elastic constants of 2D anisotropic materials, Comino and Gallego (2005) presented an inverse technique based on the Levenberg-Marquardt method by use of the boundary element method.

Huang et al., (2006) studied the optimal measurement locations for identification of elastic constants of 2D orthotropic materials by an iterative process. Lecompte et al., (2007) proposed a technique based on the Gauss-Newton optimization method for identification of the four in-plane elastic constants of an orthotropic plate, where strain measurements from a biaxial tensile test were used in their FEM computations. Bruno et al., (2008) presented an inverse method for identification of the four elastic constants of orthotropic plates with arbitrary shapes. In the computations by their genetic algorithm, the FEM was employed and the full-field measurements of surface displacements were used. Furukawa and Pan (2010) presented an energy-based technique for stochastic identification of the four elastic constants of 2D orthotropic materials. This technique recursively evaluates the unknown elastic constants at every acquisition of measurements using the Kalman filter. Hematiyan et al., (2012) presented an inverse technique based on the BEM to identify the six elastic constants of 2D anisotropic bodies using displacement measurements. Their

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formulation makes it possible to use measurement data from several experiments for identification of the elastic constants. They concluded that two or three simple experiments would result in more accurate solutions than a single complicated test. Chen et al., (2013) proposed an inverse technique based on the scaled finite boundary element method to identify the four elastic constants of 2D orthotropic bodies using measured displacements and the Levenberg-Marquardt method. They considered inequality constraints on the elastic constants in their inverse analysis. Nigamaa and Subramanian (2014) used the eigen-function virtual fields method (VFM) to identify the four in-plane elastic constants of orthotropic materials using the full-field strain data obtained from a bending test. In their study, two elastic constants were observed to have severe sensitivity to noises in the strain data. Furthermore, Jiang et al., (2015) used the VFM to identify the four orthotropic elastic constants of fiber-reinforced polymer-matrix composites using full-field strain measurements from a three-point bending test. Recently, Gu and Pierron (2016) numerically examined four types of static experiments for identification of elastic constants of 2D orthotropic materials using the VFM. Despite its success in inversely identifying 2D elastic properties, the same effectiveness of the VFM cannot be met for 3D members due to the necessary full-field measurements.

As is obvious from the literature review, all works in the past were presented for identification of 2D elastic constants only. Although 3D generally anisotropic materials with 21 elastic constants may not be of important interest for engineering applications; however, materials with one or more plane of symmetry, but with unknown orientations of planes of symmetry, may be considered as 3D generally anisotropic materials in the identification process. After determining the values of elastic constants, the orientations of material planes of symmetry or the direction of principal axes of the material can be found. To the authors' best knowledge, a method for identification of all elastic constants of 3D general anisotropic materials using static measurements has not been presented in the literature yet.

For a 2D general anisotropic material, there are six independent elastic constants, while those for 3D cases are twenty-one. For identification of the large number of elastic constants of 3D cases, the inverse analysis is very challenging indeed. It should be mentioned that if a 2D specimen is obtained by cutting a part of the main 3D anisotropic body, the specimen is not necessarily an orthotropic material with in-plane principal axes and therefore, the specimen exhibits out-of-plane deformation even if it is subjected to in-plane loads. In other words, in general, it is impossible to find some elastic constants of a 3D generally anisotropic material by conducting experiments on a 2D specimen.

In this paper, a new stable inverse method is presented for identification of all twenty-one elastic constants of 3D general anisotropic materials using static measurements. In the process of the inverse analysis, nonlinear inequality constraints on elastic constants need to be satisfied. For solving the problem more easily, the constrained inverse problem is first converted to an unconstrained one by the Cholesky factorization theorem and a suitable transformation. In the inverse analysis, the damped Gauss-Newton method (Björck, 1996; Ortega and Rheinboldt, 2000) is employed for minimization of the cost function defined by the Tikhonov regularization method (Tikhonov and Arsenin, 1977). It is very unlikely to accurately determine all the twenty-one constants using only one simple static experiment. In the present work, a multi-experiment based inverse formulation is developed for carrying out the inverse analysis. Herein, several simulated experimental data are supplied using the direct analyses of a meshless method. Necessary sensitivity analyses are accurately formulated by direct differentiation of the weak form of the governing equations. In the end, several numerical examples are presented to demonstrate the feasibility of the proposed method for the 3D inverse analysis. Additionally, the effects of measurement noise are also studied for these examples. It turns out that all the twenty-one elastic constants of a 3D arbitrarily shaped anisotropic solid can be correctly identified using several (even less than 21) sensors by employing a few static tests.

2. Basic equations of anisotropic elasticity

In this section, basic equations of anisotropic elasticity are reviewed. Original works on anisotropic elasticity have been carried out by Love (1927), Hearmon (1961) and Lekhnitskii (1981); however, basic equations of anisotropic elasticity in a modern notation can be found in standard elasticity or continuum mechanics textbooks (Sadd, 2009; Lai et al., 2009).

As is well known, the elastic deformation of an anisotropic solid is governed by the Hooke's law and the strain-displacement relations. In absence of body forces, the equilibrium condition states

$$\sigma_{ij,j} = 0, \tag{1}$$

where σ_{ij} are components of the stress tensor. The stress-strain relation of the Hooke's law is expressed as

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}, \tag{2}$$

where C_{ijkl} is the fourth-order stiffness tensor, satisfying

$$C_{ijkl} = C_{ijlk}, \quad C_{jikl} = C_{ijlk}, \quad C_{ijkl} = C_{klij} \tag{3}$$

and ε_{kl} are strain components, related to the displacements u_i by

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{4}$$

Eqs. (1)–(4) must be satisfied at every point of an anisotropic solid, either inside the domain Ω or over the boundary Γ; in addition, boundary conditions must also be satisfied. Let Γ_u and Γ_t be two non-overlapping parts of the boundary, i.e. $\Gamma = \Gamma_u \cup \Gamma_t$ and $\Gamma_u \cap \Gamma_t = 0$. Suppose the boundary conditions of the Dirichlet and Neumann types are given as follows:

$$\mathbf{u}(\mathbf{x}) = \hat{\mathbf{u}}(\mathbf{x}) \qquad \mathbf{x} \in \Gamma_u, \tag{5}$$

$$\mathbf{t}(\mathbf{x}) = \hat{\mathbf{t}}(\mathbf{x}) \qquad \mathbf{x} \in \Gamma_t, \tag{6}$$

where $\hat{\mathbf{u}}(\mathbf{x})$ and $\hat{\mathbf{t}}(\mathbf{x})$ are prescribed displacement and traction vector functions, respectively. The traction vector is related to the stress tensor as follows:

$$t_i = \sigma_{ij} n_j, \tag{7}$$

where n_j are components of the unit vector normal to the boundary. From the conditions of Eq. (3), the number of independent elastic constants of a general anisotropic solid is reduced to 21 and Eq. (2) can be written in the following matrix form:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{1111} & & & & & \\ C_{2211} & C_{2222} & & & \text{Sym.} & \\ C_{3311} & C_{3322} & C_{3333} & & & \\ C_{2311} & C_{2322} & C_{2333} & C_{2323} & & \\ C_{3111} & C_{3122} & C_{3133} & C_{3123} & C_{3131} & \\ C_{1211} & C_{1222} & C_{1233} & C_{1223} & C_{1231} & C_{1212} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{bmatrix}, \tag{8}$$

or alternatively expressed in the Voigt notation as follows:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} C_{11} & & & & & \\ C_{21} & C_{22} & & & \text{Sym.} & \\ C_{31} & C_{32} & C_{33} & & & \\ C_{41} & C_{42} & C_{43} & C_{44} & & \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}. \tag{9}$$

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