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Representative experimental and computational analysis of the initial resonant frequency of largely deformed cantilevered beams

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ABSTRACT

This paper presents an investigation of the initial resonant vibration frequency of largely deformed cantilevered beams. This study is important because the vibration of beams with this level of deformation occurs from a position that differs significantly from the undistorted configuration. As the deformation of the beam increases, the lateral force component acting tangentially to the beam's newly curved shape also increases, changing its stiffness and influencing the resonant frequency of the system. The experimental component of this study shows that as the deformation of the beam increased, the difference between the resonant frequencies of the straight and deformed beams increased with the length of the beam. However, this difference decreases and the frequency rises slightly, precisely indicating the moment in which the aforementioned lateral force component starts to influence the stiffness of the bending piece. A computer model was then prepared to obtain the resonant frequency of the beam using the finite element method to confirm the experimental results for two scenarios: considering the beam in its horizontal configuration, and considering the beam in its deformed configuration using nonlinear static analysis processing, which has been previously applied in other theoretical investigations of large displacements. The results of the second method demonstrated a favorable approximation of the experimental values. An analytical evaluation was also performed.

1. Introduction

The vibration response of framed structures modeled as beams and columns has been and continues to be studied extensively because these beams and columns are fundamental structural elements in many industrial applications. These structures are frequently subjected to dynamic loading, resulting in large amplitude deformation and vibrations that introduce a geometric type of nonlinearity influencing the dynamic behavior of the structure.

The Euler–Bernoulli beam represents a continuous structural member, and its vibrations are governed by nonlinear partial differential equations for which exact analytical solutions cannot be found (Awrejcewicz et al., 2015; Lee, 2002). The dynamic bending of beams, also known as the flexural vibration of beams, was first investigated by Daniel Bernoulli in the late 18th century. Bernoulli's equation of motion of a vibrating beam tended to overestimate the resonant frequencies of beams and was marginally improved by Rayleigh (1945) with the addition of a mid-plane rotation. The nonlinear vibrations of Euler–Bernoulli beams have been studied for a long time, since approximately 1750 (Rayleigh, 1945). The work presented by Timoshenko (1921) improved the theory further by identifying the effect of shear on the dynamic response of bending beams. This contribution was expounded upon by Kaneko (1975) and Rosinger and Ritchie (1977), extending the theory to problems involving high-frequency vibrations, for which the dynamic Euler–Bernoulli theory is inadequate. Nonetheless, the Euler–Bernoulli and Timoshenko theories for the dynamic bending of beams continue to be widely used by engineers Fey et al. (2011) highlight the study performed by Stoykov and Ribeiro (2011), which investigated the geometric nonlinear periodic vibrations of beams under harmonic forces, conducting a set of numerical experiments modeling the movement of a clamped–clamped aluminum beam.

However, it is important to consider that dynamic loading can have significant effects on the responses of structural characteristics. In many applications, nonlinear large deformation beam elements are needed to account for bending, axial, and shear deformation properties (Nachbagauer et al., 2011).

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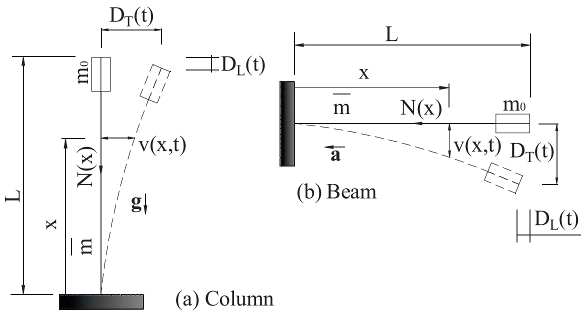

Fig. 1. Diagram of linear problem.

This article presents an experimental study of the initial resonant vibration frequency of largely deformed cantilevered beams that remain under the influence of their geometric stiffness. The experimental results are compared with those determined analytically from a closed-form equation and those obtained via the finite element method (FEM), using modal analysis for eigenvalues obtained for the modified stiffness of the beam after large static displacement. Specifically, this work seeks to identify the initial resonant frequency in cantilevered beams with large deformations that are caused by geometric effects or by variation in the beam's stiffness resulting from a transverse force component acting tangentially to the beam axis. This transverse force component must include the beam's own weight, which generates a certain amount of tension at each different beam length.

2. Linear vibration of a cantilever beam/column

To understand the problem of small amplitude vibrations in beams/columns, the configuration presented in Fig. 1 is considered. Notably, the system containing just the lateral degree of freedom has been in an undamped free movement. This system is composed of a prismatic bar made from a linear elastic material, which is embedded in the left side or in the base bearing its own weight, and a mass on the free extremity. The Euler–Bernoulli hypothesis, in which the cross section was normal to the axis of the beam before deformation remains straight after deformation, inextensible, and rotate as rigid lines to remain to perpendicular the bent axis, is considered. For the linear case, the movement of the system does not alter the orientation of the generalized normal force *N(x)*, which has to be taken into consideration. A similar mathematical procedure is presented by Clough and Penzien (1993). If the beam/column is in movement, the amplitude of the lateral displacement of the free end of the beam/column is given by the generalized coordinate, represented by $D_T(t)$. Thus, any transverse displacement of the bar is unequivocally defined by

$$v(x,t) = \phi(x) D_T(t)\ , \tag{1}$$

where ϕ(x) is a shape function that satisfies the boundary conditions of the problem and $D_T(t)$ is a transient displacement.

The use of Eq. (1) implies the constancy of the shape of vibration with respect to time, representing that only the amplitude of the movement is varying, and it varies harmonically with the free-moving condition. The assumption of *f* as a function of *x* effectively restricts the bar to a system with a single degree of freedom (SDOF). This affirmation is based on the Rayleigh method (1945) that assumed that a system with infinite degrees of freedom can be replaced by a single degree of freedom (SDOF) system in order to approximate its frequency. The Rayleigh method depends entirely on the functional form that is used to represent the free vibration mode. If the exact shape were assumed, the exact corresponding frequency would be generated by this method. In fact, the most critical aspect of this method is the selection of appropriate functions. If these functions form a complete set, the natural frequencies converge to the actual value. If still, considering these assumptions, the principle of virtual work is used at the same time a single close-form formulation is found.

Thus, the frequency of vibration can be used to find the maximum strain energy developed during the movement and the maximum kinetic energy. Therefore, the principle of virtual work is sufficient to describe the movement of the structure.

The work done by the external forces over the virtual displacement is

$$\delta W_E = -\int_0^L f_I(x,t)\,\delta v(x)\,dx + N(x)\,\delta D_L\ , \tag{2}$$

where $f_I(x,t) = \bar{m}(x)\ddot{v}(x,t)$ represents the inertial force and D_L is the axial displacement. The work of the virtual internal forces is given by

$$\delta W_I = \int_0^L M(x,t)\,\delta v''(x)\,dx\ , \tag{3}$$

where $\delta v''(x) = \frac{\partial^2 \delta v(x)}{\partial x^2}$.

To find the axial displacement $D_L(t)$, it is necessary to take an infinitesimal element of the elastic line of the bar. Then, the shortening of the axis due to the axial displacement will be

$$ds - dx = \sqrt{dx^2 + dv^2} - dx = dx\sqrt{1 + \left(\frac{dv}{dx}\right)^2} - dx\ . \tag{4}$$

Because the superior order terms are small when compared to unity, an acceptable approximation by the binomial expansion yields

$$ds - dx = dx\left[1 + \frac{1}{2}\left(\frac{dv}{dx}\right)^2\right] - dx = \frac{1}{2}\left(\frac{dv}{dx}\right)^2. \tag{5}$$

By integrating Eq. (5) over the length of the entire beam, the following equation is obtained, with the first derivate indicated as a superior line on the right side:

$$D_L = \frac{1}{2}\int_0^L \left[v'(x,t)\right]^2 dx\ . \tag{6}$$

Because the parameters necessary for the solution of the problem may be expressed as functions of the generalized coordinate D_T and a form function ϕ(x), the following equations are obtained:

$$\begin{array}{lll} v(x,t) = \phi(x) D_T(t); & \ddot{v}(x,t) = \phi(x)\ddot{D}_T(t); & \delta v'(x,t) = \phi(x)'\delta D_T(t); \\ v'(x,t) = \phi'(x) D_T(t); & \dot{v}''(x,t) = \phi''(x)\dot{D}_T(t); & \delta v''(x,t) = \phi(x)''\delta D_T(t). \\ v''(x,t) = \phi''(x) D_T(t); & \delta v(x,t) = \phi(x)\delta D_T(t); & \delta D_L = \int_0^L v'(x,t)\,\delta v'(x)\,dx \end{array} \tag{7}$$

Conveniently, substituting terms defined in Eq. (7) into Eqs. (2) and (3) yields, respectively,

$$\delta W_E = \left[-\ddot{D}_T(t)\int_0^L m_1(\phi(x))^2 dx + D_T(t)\int_0^L N(x)(\phi'(x))^2 dx\right]\delta D_T, \tag{8}$$

and

$$\delta W_I = \left[D_T(t)\int_0^L EI\big(\phi''(x)\big)^2 dx\right]\delta D_T\ . \tag{9}$$