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Modelling and experiments of glassy polymers using biaxial loading and digital image correlation

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ABSTRACT

Experimental data are needed to evaluate constitutive models. The richer the experimental data, in terms of different deformation modes for example, the better the constraints on the model. To this end, the mechanical response of glassy polycarbonate (PC) is studied using biaxial tension experiments with novel sample geometries. Deformation fields are measured using full-field digital image correlation (DIC) to reveal a large difference in the strain localisation behaviour of the material depending on the amount of lateral deformation. The difference in the localisation behaviour is also reflected in the macroscopic force-displacement response. The experimental data acquired in the experiments are used to examine the ability of a physically-motivated constitutive model to predict the mechanical response of the material during biaxial deformation. To improve the numerical predictions, the elastic part of the constitutive model is modified such that the initial non-linear response due to volumetric deformation is accurately captured. This new elastic model is motivated by the experimental results, which show that the commonly used quadratic form of the elastic free energy results in a too stiff response during biaxial tension.

1. Introduction

Polycarbonate (PC) is an amorphous polymer with a relatively high glass transition temperature, a high impact strength and good optical properties. Due to these favourable features, PC is often used in industrial and consumer products such as safety glass, machine guards and containers. Glassy amorphous polymers, such as PC at room temperature, are commonly used as load carrying components in which the material will likely be subjected to multiaxial loading conditions and complex deformation histories. The understanding of, and the ability to predict, the mechanical properties and the evolution of the material under mechanical load is, therefore, of great importance during the design process of such components.

Over the years, a considerable amount of work has gone into refining the constitutive models for predicting the mechanical behaviour of glassy polymers. Many models make use of non-Gaussian chain statistics to represent the macromolecular network. Boyce et al. (1988) proposed a 3-chain model to represent the polymer network using the non-Gaussian statistical model by Wang and Guth (1952). As the 3-chain model was unable to accurately distinguish between different states of deformation, Arruda and Boyce (1991, 1993) proposed a model using eight chains to represent the polymer network that better captures the mechanical behaviour found experimentally. In the 8-chain model, the chains extend from the centre to the corners of a unit cube. Wu and van der Giessen (1993) showed that the three- and eight-chain models can represent an upper and lower bound for the network stiffness, respectively. Motivated by this, Wu and van der Giessen (1993) proposed a linear combination of the two models. In the same paper, they also used the full-network model by Treloar and Riding (1979) to model the response of glassy PC under three-dimensional loading. The full-network model uses a chain orientation distribution function (CODF) to distribute a large number of chains during deformation. Later, Harrysson et al. (2010) proposed a model capable of having a non-affine evolution of the microstructure using a CODF.

Many amorphous solids exhibit a non-linear response at small strains prior to a stress peak. Different approaches have been suggested to predict the smooth, pre-peak transition. Hasan and Boyce (1995) developed a one-dimensional framework for the viscoplastic flow of glassy polymers, using a set of internal state variables to describe the evolution of the microstructure. The flow theory by Hasan and Boyce, which is based on evolution of free volume, is able to predict a smooth pre-peak transition. This flow theory was later implemented in a three-dimensional setting by Miehe et al. (2011). Anand and Gurtin (2003) used a single internal state variable related to the local free volume to capture the non-linear

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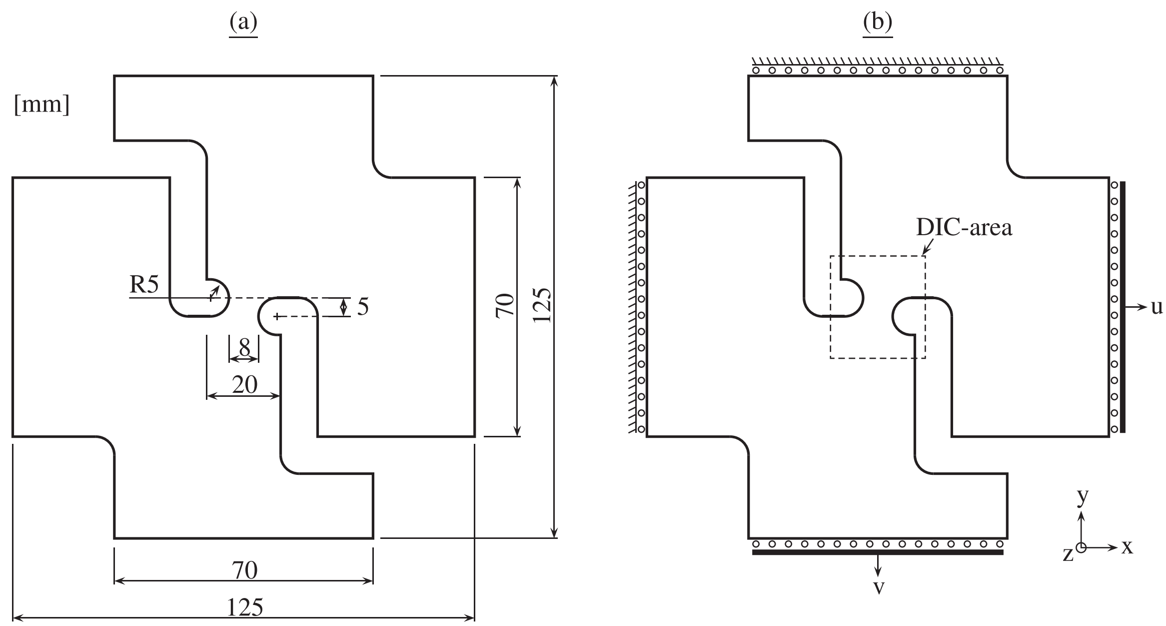

Fig. 1. (a) The dimensions of the specimen used in the biaxial deformation experiments and (b) the boundary conditions during loading and simulation. The area where the deformation is measured using DIC is indicated in (b).

pre-peak behaviour. Chowdhury et al. (2008) used a different approach to improve the small strain behaviour by splitting the variable related to the strain softening, introduced by Boyce et al. (1988), into two parts, which results in a smooth transition from the elastic to the plastic response compared with the original model.

When characterising the mechanical response of polymers, the focus has often been on the macroscopic stress-strain response from uni-axial or multiaxial compression tests where the deformation is measured at the boundaries of the specimen, or using an extensometer, cf. e.g. Arruda et al. (1995), Dreistadt et al. (2009) and Ames et al. (2009). For such macroscopic responses to be representative, the deformation must be homogeneous within the gauge length, i.e. necking, barrelling or buckling are not allowed. When the deformation is inhomogeneous, the need for full-field measurement techniques that provide displacement measurement at a large number of measurement points is evident. One readily available full-field technique is digital image correlation (DIC) where the deformation is measured by tracking the motion of pixel subsets in images taken of a specimen at different stages of deformation, cf. Parsons et al. (2004, 2005), Grytten et al. (2009) and Poulain et al. (2013). As full-field measurements can be applied to experiments performed under inhomogeneous conditions, the outcome of such experiments can provide richer information about the material behaviour. By performing experiments under multiaxial or inhomogeneous deformation conditions, a few tests can be used to identify large sets of constitutive parameters, cf. Hild and Roux (2006) and Avril et al. (2008). Multiaxial loading can be achieved in several ways, such as: multiaxial compression (e.g. Ravi-Chandar and Ma, 2000); using tube shaped specimens and combining pressure with axial or torsional loads (e.g. Hu et al., 2003); biaxial tension loading (e.g. Chevalier et al., 2001; Johlitz and Diebels, 2011).

Various approaches, many of which use nominally pure deformation modes, have been used to evaluate the performance of constitutive models. Tests performed under uni-axial, simple shear or plane strain conditions, where the experimental data consists of the macroscopic response, have been used by, e.g., Boyce and Arruda (1990), Tomita (2000), Harrysson et al. (2010) and Holopainen and Wallin (2013) to validate the performance of their respective models. As has been discussed above, full-field methods should be utilised to capture inhomogeneous deformation experimentally. DIC has been used in combination with uni-axial tension tests of dog-bone shaped or notched specimens to evaluate constitutive models in the works of, e.g., Miehe et al. (2009) and Engqvist et al. (2016b). By comparing the simulated response to measured deformation fields, a more comprehensive evaluation of the model is possible, as not only the macroscopic response of the structure is available for comparison but also local variations of the deformation can be studied. While a comparison of simulations and full-field measurements of uni-axial tensile experiments provides the ability to constrain inhomogeneous deformation in terms of necking and neck propagation, it does not, in general, include much information about the influence of shear or volumetric deformation. To this end, more general deformations have been studied by e.g.: combined tensile and shear deformation of glassy PC, by Holopainen and Wallin (2013), biaxial tensile loading of silicone rubber, by Johlitz and Diebels (2011) and creasing of paperboard, by Borgqvist et al. (2015).

An issue when developing simulation models for more complex deformation scenarios is the general lack of experimental data, since this often requires non-standard test equipment. To this end, biaxial tensile experiments of glassy PC have been conducted within this work. The deformation of the specimen is measured using full-field 3D-surface DIC. The experimental data are used to explore, and improve, the ability of the physically-motivated constitutive model by Engqvist et al. (2016b) to predict the mechanical response of glassy PC.

2. Preliminaries

Referring to the coordinate system defined in Fig. 1, the *x*, *y* and *z* directions are denoted *lateral, axial* and *out-of-plane* or *thickness* direction, respectively. Second order tensors and vectors are denoted by bold-face Roman letters and the second order unit tensor is denoted by $\mathbf{1}$. The transpose and the inverse of a second order tensor are denoted as $[\cdot]^T$ and $[\cdot]^{-1}$, respectively. The superscript $[\cdot]^{\mathrm{dev}}$ denotes the deviatoric part of a second order tensor, defined as $[\cdot]^{\mathrm{dev}} = [\cdot] - \frac{1}{3}\mathrm{tr}[\cdot]\mathbf{1}$ where $\mathrm{tr}[\cdot]$ denotes the trace of the tensor. The symmetric part of a second order tensor is denoted as