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A new energy based mixed-mode cohesive zone model

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ABSTRACT

In this work a novel energy based mixed-mode I+II cohesive zone model is proposed. The model uses the incremental calculation of energy dissipated at each integration point in order to define the damage parameter that simulates gradual material softening as a function of loading. The new model was compared with classical mixed-mode cohesive zone models in which the damage parameter is determined from relative displacements. With this aim, simulations of the Mixed-Mode Bending test considering carbon-epoxy bonded joints were used to assess the differences between the two methods. It was shown that this test reveals a constant global mode ratio with important variations of local mode ratio. The mode I and II fracture energy components issuing from the results were compared with the energy based criterion used as input. It was verified that the relative displacement based method reveals inconsistent results when different pure mode cohesive properties are considered as a consequence of local mode ratio variation. In contrast, it was concluded that the proposed energy based method behaves well in several different circumstances, thus constituting a better solution to deal with general mixed-mode fracture problems.

1. Introduction

The use of cohesive zone models (CZM) in fracture mechanics problems has been increasing in recent years. These models are able to deal with simulation of onset and non-self-similar damage growth. The consideration of an initial crack is not required and damage propagation occurs without user intervention, i.e., crack initiation and growth are integrated in the same model. Stress based criteria are frequently utilized to deal with damage onset and energy fracture mechanics based criteria are utilized to simulate the damage growth (Camanho et al., 2003). The occurrence of damage is mimicked by a softening relationship between tractions and relative displacements at a given integration point, thus simulating gradual material degradation. In general, CZMs are implemented in interface finite elements (Mi et al., 1998; Petrossian and Wisnom, 1998; de Moura et al., 2000) positioned between solid elements at the critical planes where damage is expected to arise, which are easily identified in the bonded joints case. In fact, in this type of connections damage propagation is usually confined to planes close to or at the interfaces between adhesive and adherends or inside the adhesive, thus making the application of CZM an appealing design concept strategy.

The development of CZM for pure mode loading cases is well established and several different laws can be found in the literature. The most used owing to its simplicity is the bilinear cohesive law (Mi et al., 1998; Camanho et al., 2003). The trilinear law with bilinear softening relationship is also widely used to deal with several damage mechanisms as it is the case of micro-cracking and fibre-bridging observed in interlaminar fracture of composites (Tamuzs et al., 2001), wood (Dourado et al., 2008) and bone (Pereira et al., 2012). Polynomial and exponential laws were proposed by Needleman (1987) and Xu and Needleman (1994), respectively. Tvergaard and Hutchinson (1993) developed a trapezoidal law to deal with plasticity inherent to the fracture process. Williams and Hadavinia (2002) and Alfano (2006) analysed the influence of the cohesive law shape on the analysis of debonding considering pure mode problems. In both cases the authors have concluded that the shape of the cohesive law has a small influence on the results obtained. However, real structures rarely behave under pure mode loading thus rendering pure mode models of limited interest. Actually, general mixed-mode loading is present in the majority of real applications which makes the development of appropriate mixed-mode CZM essential. For example, in bonded joints the crack is usually constrained to grow along

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a pre-established path which naturally occurs under mixed-mode loading due to a combination of shear and normal stresses.

Mixed-mode CZM have been proposed to deal with general loading namely the combination of mode I and II loading case. Gonçalves et al. (2003) developed a mixed-mode I+II CZM to simulate the debonding process of aluminium single-lap joints loaded in tension. The model is based on the definition of two surfaces: the Initial Damage Surface (IDS) defined by the critical onset relative displacements and the Final Damage Surface (FDS) defined by the ultimate relative displacements. Up to IDS there is a linear elastic relation between tractions and relative displacements. Between IDS and FDS, a linear softening process defined by the position of the current damage surface is assumed, thus simulating material softening. For each loading mode component a damage parameter ranging between zero and unity was defined. It was observed that these parameters do not reach simultaneously the unity and it was assumed that when one of them reaches it the other was also automatically set to one. In order to overcome this drawback equivalent relative displacements based models were developed (Mi el al., 1998; Camanho et al., 2003 and Durão et al., 2006). In the bilinear models the damage parameter is calculated from the critical onset and ultimate equivalent relative displacements obtained by the square root of the sum of squared shear and normal components. These critical displacements depend on the current mode mixity which is commonly identified by the ratio between the displacement components. In cases characterized by quite different pure mode cohesive properties and mode ratio variation during loading, the values of those critical displacements vary for a given integration point along the softening process leading to inconsistencies on the estimation of the damage parameter. Turon et al. (2006) verified that these models do not work properly giving rise to wrong estimations of the damage state during softening. To overcome this drawback, those authors proposed a damage model for the simulation of delamination under variable mode ratio based on the definition of damage onset and propagation surfaces using the Benzeggagh and Kenane (1996) energy based fracture criterion. The authors argue that their formulation provides a smooth transition for all mixed-mode ratios between the initial and the propagation damage surfaces defined from stress and energy based criteria, respectively. The model avoids the restoration of the cohesive state and guarantees a positive energy dissipation under variable mode ratio. Dimitri et al., (2015) analysed the effect of the coupling parameters on cohesive stress-interface relative displacement behaviour, energy dissipation and mixed-mode failure domains corresponding to different loading paths for some widely used exponential and bilinear mixed-mode CZMs. The authors concluded that all the models analysed, with one exception, produce inconsistencies at the local level. Examples of those inconsistencies are non-convex cohesive stress-relative displacement curves, loading path-dependent failure domains and improper accounting for energy dissipation.

In this work a novel mixed-mode I+II damage model is proposed. The model is based on incremental assessment of the energy dissipated at each integration point during the softening process which is used to calculate the damage parameter. Following this procedure the energy is accurately estimated even when drastic variations of mode ratio occur in problems characterized by large differences between the pure mode cohesive properties. The Mixed-Mode Bending test on carbon-epoxy bonded joints was used to perform a comparison between the proposed model and the classical cohesive zone model based exclusively on relative displacements. Important conclusions were drawn about the model performance under various loading conditions and material properties influencing the mode ratio during propagation.

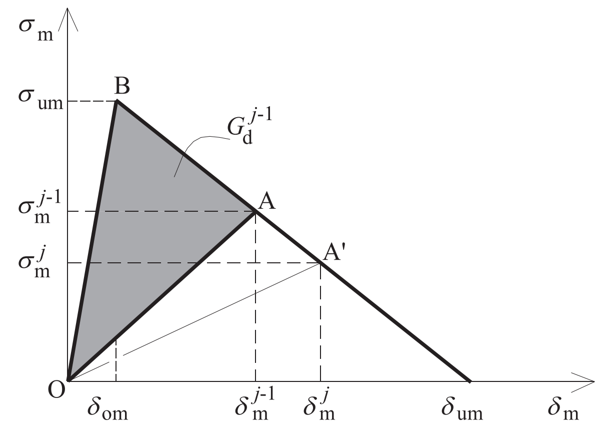

Fig. 1. Mixed-mode I+II cohesive law with linear softening relationship between stresses and relative displacements.

2. Displacement based CZM

Cohesive zone models (CZM) consist in a stress ($\boldsymbol{\sigma}$) versus relative displacements ($\boldsymbol{\delta}$) softening relationship based on the following equation

$$\boldsymbol{\sigma} = [\mathbf{I} - \mathbf{D}]\,\mathbf{E}\,\boldsymbol{\delta} \tag{1}$$

where **I, D** and **E** are the identity, damage and initial stiffness diagonal matrices, respectively. Matrix **E** contains the initial stiffness parameter (k) which is defined as being the largest value that does not induce numerical problems and avoids spurious interpenetrations (Durão et al. 2006). The diagonal matrix **D** contains the damage parameter d defined from the linear softening relationship (i.e., between δ_{om} and δ_{um} in Fig. 1)

$$d = \frac{\delta_{um}(\delta_m - \delta_{om})}{\delta_m(\delta_{um} - \delta_{om})} \tag{2}$$

where the subscript "m" refers to the equivalent mixed-mode displacement and δ_{om}, δ_{um} and δ_m represent the onset, ultimate and the maximum relative displacements reached during loading history, respectively. Damage onset occurs when the quadratic stress criterion

$$\left(\frac{\sigma_I}{\sigma_{uI}}\right)^2 + \left(\frac{\sigma_{II}}{\sigma_{uII}}\right)^2 = 1 \tag{3}$$

is satisfied. In this equation the numerators are the stress components of the mixed-mode loading that lead to damage initiation, while the denominators correspond to limit values under pure modes. It is usually assumed that normal compressive stresses do not induce damage which means that the first term of Eq. (3) is neglected under these conditions. Hence, considering the following relations

$$\delta_m = \sqrt{\delta_I^2 + \delta_{II}^2}; \qquad \beta = \frac{|\delta_{II}|}{\delta_I} \tag{4}$$

the equivalent mixed-mode displacement corresponding to damage onset becomes (combining Eqs. (3) and (4))

$$\delta_{om} = \delta_{oI}\delta_{oII}\sqrt{\frac{1+\beta^2}{\delta_{oII}^2 + \beta^2\delta_{oI}^2}} \tag{5}$$

The linear energy based criterion was used to deal with damage propagation assuming $\gamma = 1$ in the power law energy based relation

$$\left(\frac{G_I}{G_{Ic}}\right)^\gamma + \left(\frac{G_{II}}{G_{IIc}}\right)^\gamma = 1 \tag{6}$$

The strain energy in each mode is given by

$$G_i = \frac{1}{2}\sigma_{um,i}\,\delta_{um,i} \quad \text{with } i = \text{I, II} \tag{7}$$