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Global and local interactive buckling behavior of a stiff film/compliant substrate system

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ABSTRACT

This paper elucidates the global and local interactive buckling behavior of a stiff film resting on a compliant substrate under uniaxial compression. The resulting governing non-linear equations (non-autonomous fourth-order ordinary differential nonlinear equations with integral conditions) are then solved by introducing a continuation algorithm, which offers considerable advantages to detect multiple bifurcations and trace a complex post-buckling path. The critical conditions for local and global buckling and respective post-buckling equilibrium paths are carefully studied. Two different evolution mechanisms of buckling modes and processes from destabilization to restabilization (snap-back) are observed beyond the onset of the primary sinusoidal wrinkling mode in the post-buckling range. In addition, the shear modulus of an orthotropic substrate acts as a dominant role in the bifurcation portrait. Our results offer better understanding of the global and local buckling behaviors of such a bilayer system, and can open up new opportunities for the design and applications of novel nanoelectronics.

1. Introduction

Bilayer systems consisting of a stiff thin film on a compliant substrate have provoked a surge of research interest in academic domains over the last decade (Bowden et al., 1998; Huang and Suo, 2002; Schweikart and Fery, 2009). When such a system is subjected to a large compression, it may lose its original flat surface and leads to buckling, which may dramatically alter their inherent structural equilibrium and thus results in a series of changes in properties (Efimenko et al., 2005; Koch et al., 2009; Wang et al., 2013a). As such, the film/substrate system in the post-buckling state has potential uses as stretchable electronic devices, tunable diffraction and phase gratings or patterned platforms for cell adhesion (Harrison et al., 2004; Stafford et al., 2004; Rogers et al., 2010).. Many previous studies have shown that a number of possible post-buckling morphologies may occur in the surface, including global buckling, sinusoidal, checkerboard, herringbone, etc (Chen and Hutchinson, 2004; Cai et al., 2011; Wang et al., 2013b; Xu et al., 2015). How to comprehensively evaluate these possible morphologies, as well as their relationship and formation mechanism, remains a challenge (Li et al., 2012).

Thus far, several theoretical approaches, such as linear perturbation analysis and non-linear buckling analysis, have been proposed and become effective means for exploring the instability behavior of the systems (Wang et al., 2008; Im and Huang, 2008; Zhuo and Zhang, 2015). However, most of these studies have focused on the critical load and morphologies at the initial stage of instability threshold. There is a lack of investigation on the morphological evolution and mode transition in the post-buckling stage due to incredibly complication, such as geometrical and material nonlinearities, loading path dependence, etc. Therefore, reliable numerical solution techniques for tracing and branch switching post-buckling response of film/substrate system are in demand.

Recent efforts have been devoted to such post-buckling analysis by using finite element methods (Sun et al., 2012; Cao et al., 2012), which is more flexible to describe complicated geometries and boundary conditions. However, the simulation may not capable to trace a more complex case, especially whose post-buckling path is accompanied with snap-back or snap-through phenomenon due to the presence of secondary instabilities such as local buckling. Information about bifurcations is not immediately available to the user and stopping and restarting the simulation at a fixed point is not straightforward (Pirrera et al., 2010; Ke et al., 2016).

For all of the aforementioned reasons, we adopted a continuation algorithm to solve the resulting non-linear equations, in consideration of possible bifurcations along the equilibrium path

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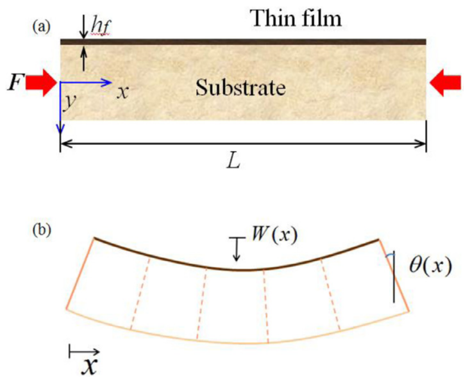

Fig. 1. (a) Illustration of a film/substrate system under compression; (b) Sway and tilt components.

tracing of the non-linear response of a system. It appears as a significantly efficient path-following technique with two key features, the detection of bifurcation points and the branch-switching from one equilibrium path to another bifurcation path, and has been demonstrated to be an efficient technique to deal with various non-linear problems in solid mechanics (Damil and Potier-Ferry, 1990; Wadee and Hunt, 1998; Abichou et al., 2002; Wadee et al., 2015). The following problem thereby can be computed by coupling analytical models with the continuation algorithm.

In this paper, a comprehensive study of the global and local instability behaviors in film/substrate systems is provided from a continuum mechanics perspective. The study is structured as follows. In Section 2, the buckling governing equations, taking account of the global and local deformations are established. The aim here is to describe different morphologies of the film/substrate under compression. In Section 3, the resulting governing equations are solved by introducing a continuation algorithm to trace the whole equilibrium paths and obtain post-buckling characteristics of the systems. Section 4 presents the results, including the critical condition for local and global buckling, the respective post-buckling equilibrium paths and mode evolutions. Finally, the effects on bifurcation portrait of orthotropic substrate are discussed.

2. Model formulation

In this work we investigate the instability behavior of an isotropic stiff film with a thickness of h_f on a soft orthotropic substrate. The compliant substrate has different Young's moduli in the axial direction Ex and the transverse direction Ey and an associated shear modulus G with a finite thickness of h_s and a length of L_x. Poisson's ratios in the x- and y-directions are vx and vy, respectively. The structural parameters are described by a Cartesian coordinate system, where the longitudinal direction and the transverse horizontal direction are determined as the x-axis and the y-axis, respectively (Fig. 1(a)). The bilayer system deforms in the x-y plane and distributes uniformly along z-axis (perpendicular to the x-y plane). A steady axial compressive force F is applied at each side of the substrate.

Different instability phenomena may occur in this bilayer system, including global buckling, local wrinkling and their interactive buckling. In order to describe these complicated instability phenomena, the formulation of the model is described through both the global and the local modal displacements. Firstly, two dimensionless factors are introduced to present the global sway and tilt, i.e., q_s and q_t, directly to describe the shear effect (Fig. 1(b)), which has a great effect on the formation of local buckling.

Here q_s not necessarily equals q_t, which is different from the Euler model where the two are assumed to be identical. Then, the global sway $W(z)$ and tilt $\theta(z)$ can be approximated as the following expressions (Bai and Wadee, 2015; Wang et al., 2016):

$$W(x) = q_s L \sin\frac{\pi x}{L}$$
$$\theta(x) = q_t \pi \cos\frac{\pi x}{L}. \tag{1}$$

Thus the corresponding shear strain is given by:

$$\gamma_{xy}(x) = (q_s - q_t)\pi \cos\frac{\pi x}{L}. \tag{2}$$

Secondly, local wrinkling deformation is described by two functions $u_f(x)$ and $w_f(x)$, for the local in-plane and transverse displacements, respectively. It is noted that these functions have no phenomenological assumptions and are sought as solutions from minimization of total potential energy. Therefore, they can well describe the buckling evolution without wrinkling pattern or number restriction.

The total potential energy of the system, Π, is mainly composed of bending energy, U_B, membrane energy, U_M, elastic strain energy of the substrate, U_S, and work done by load, U_L, and is expressed as follows:

$$\Pi = U_B + U_M + U_S - U_L. \tag{3}$$

The bending energy of the thin film is due to the collective effect of global and local deformations, and can be expressed as:

$$U_B = \frac{1}{2}EI\int_0^{L_x}(\ddot{W}^2 + \ddot{w}_f^2)\mathrm{d}x$$
$$= \frac{1}{2}EI\int_0^{L_x}\left(2q_s^2\frac{\pi^4}{L^2}\sin^2\frac{\pi x}{L} + \ddot{w}_f^2\right)\mathrm{d}x. \tag{4}$$

where $EI = \frac{E_f L_z h_f^3}{12(1-v_f^2)}$ is the flexural rigidity of the film; L_z denotes the breadth of the film; E_f and v_f are Young's modulus and Poisson's ratio, respectively. In addition, the notation "dot" above the variables denotes a spatial derivatived/dx.

Along with bending energy, the film is also subjected to membrane action. When nonlinear large deformation is taken into account, the total membrane energy can be expressed as follows according to the von Kármán hypothesis:

$$U_M = \frac{1}{2}E_f h_f L_z\int_0^{L_x}(\varepsilon_x)^2\mathrm{d}x$$
$$= \frac{1}{2}E_f h_f L_z\int_0^{L_x}(\frac{1}{2}h_s\dot{\theta} - \Delta + \dot{u} + \frac{1}{2}\dot{w}^2)^2\mathrm{d}x. \tag{5}$$

where $\frac{1}{2}h_s\dot{\theta}$ is the axial strain term corresponding to the global buckling, $\dot{u} + \frac{1}{2}\dot{w}^2$ is the axial strain term corresponding to the local wrinkling deformation, Δ is the axial strain term corresponding to purely uniform compressive strain, which contributes to the pre-buckling equilibrium path.

As the system generally has an extremely large ratio of Young's modulus ($E_f/E_s \approx 10^5$), the terms for the axial strain energy in the substrate are assumed to be small compared to the membrane energy in the film. Therefore, the substrate provides only a small proportion of the axial resistance, but the main resistance to local transverse displacement and shear deformation (Wadee and Hunt, 1998; Audoly and Boudaoud, 2008). Further justification will