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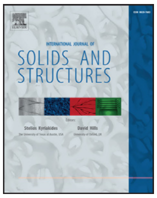

Influence of a mobile incoherent interface on the strain-gradient plasticity of a thin slab

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ABSTRACT

A thermodynamically consistent theory of strain-gradient plasticity in isotropic solids with mobile incoherent interfaces is developed. The gradients of plastic strain are introduced in the yield functions, both of the bulk and the interface, through suitable measures of material inhomogeneity; consequently, two internal length scales appear in the formalism. The rate-independent associative plastic flow rules, as proposed in the framework, are coupled with the kinetic law for interface motion. The theory is used to study plastic evolution in a three-dimensional, semi-infinite, thin slab of isotropic solid with a planar incoherent interface. The average stress-strain curves are plotted for varying length scales, mobilities, and average strain-rates. The effect of slab thickness and the two internal length scales on the hardening behavior of the slab is investigated. For all the considered cases, the stress-strain curves have two distinct kinks, indicating yielding of the bulk and at the interface. Moreover, once the interface yields, and is driven to move, the curves demonstrate both softening and rate-dependent response. The softening behavior is found to be sensitive to interface mobility and average strain-rates. These observations are consistent with several experimental results in the literature.

1. Introduction

Resistance to plastic flow in the presence of interfaces and external boundaries leads to inhomogeneous strain distribution, such as that accommodated by the geometrically necessary dislocations (Nye, 1953; Ashby, 1970). The large gradients in plastic strain near the interfaces and boundaries strongly influence the effective hardening of the solid (Aifantis et al., 2006; Fleck and Willis, 2009). The role of surfaces is in particular significant in solids with micron and sub-micron characteristic length scales (Sutton and Balluffi, 2003). Strain-gradient plasticity theories (Fleck and Willis, 2009; Fleck and Hutchinson, 1997; Gao et al., 1999; Huang et al., 2000; Gurtin and Anand, 2005) have been successfully used to study plastic deformation in solids with interfaces such as those containing an elastic-plastic boundary (Gudmundson, 2004; Fredriksson and Gudmundson, 2007; 2005; Polizzotto, 2009), grain boundaries (Voyiadjis et al., 2014; Al-Rub, 2008; Aifantis et al., 2006; Aifantis and Willis, 2005; Wulfinghoff et al., 2013; Kochmann and Le, 2008; Zhang et al., 2014b), internal surfaces in composites (Fredriksson et al., 2009), phase boundaries (Mazzoni-Leduc et al., 2008; Pardoen and Massart, 2012), and interfaces between laminates (Wulfinghoff et al., 2015). Dislocation dynamics based theories have also been used to study the effects of interfaces on plastic flow (Shu and Fleck, 1999; Puri et al., 2011; Balint et al., 2008; Zhang et al., 2014b). All these investigations are however restricted to stationary interfaces. On the other hand, various experiments (Gorkaya et al., 2011; Morris et al., 2007; Chen and Gottstein, 1988; Gourdet and Montheillet, 2002; Rupert et al., 2009; Winning et al., 2001) in polycrystalline materials have suggested that interfaces remain mobile during plastic deformation, accommodating a part of the accumulated strain near the boundaries and thereby relaxing the stresses in the body. Moreover, grain boundary propagation results in grain coarsening (increase in average grain size) in polycrystalline materials, and in this process the solids have been observed to undergo strain softening (Morris et al., 2007; Chen and Gottstein, 1988; Gourdet and Montheillet, 2002). It is therefore important to formulate a plasticity model where plastic evolution, both in the bulk and at the interfaces, is coupled with interfacial kinetics. This is precisely the purpose of the present paper. In particular, we elaborate the nature of our theory through a detailed example of a plastically deforming thin slab containing a mobile interface.

The starting point of our work is a thermodynamical framework, recently proposed by the authors (Basak and Gupta, 2015a; 2015b; 2016), using which we derive a physically motivated three-

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dimensional (3D) small deformation strain-gradient plasticity theory for isotropic solids. We allow for an energetic incoherent planar interface, whose incoherency is related to the presence of defects/inhomogeneities, to propagate within the quasi-statically deforming solid. There are two novel features of our plasticity theory. First, we assume the yield function in the bulk to depend on the incompatibility tensor (Krishnan and Steigmann, 2014; Basak and Gupta, 2016), given in terms of second order gradient of plastic strain, and the yield function on the interface to depend on an analogous incompatibility measure. Such dependence, with which we introduce two internal length scales, and which is the simplest that one could use to characterize inhomogeneity in isotropic solids (Noll, 1967), is the only source of non-locality, and hence size dependence, in our formalism. Second, we propose plasticity flow rules which are coupled with interfacial kinetic laws, so that we can model the effects of interface propagation on the overall plastic behavior of the solid. The rate-independent plasticity flow rules are assumed to be associative. The rate-dependency in the model is introduced through a linear kinetic law of interface motion. We should remark here that our strain-gradient plasticity framework, even without considering interfacial effects, is distinct from those which necessarily incorporate higher order stresses (Fleck and Willis, 2009; Gao et al., 1999; Huang et al., 2000; Gurtin and Anand, 2005; Gudmundson, 2004) or those which have a first order strain-gradient dependency in the yield function (Acharya and Bassani, 1996; Bassani, 2001). Moreover, introduction of strain gradients in our theory is through the incompatibility tensor, which is a natural measure of material inhomogeneity distribution in isotropic solids (Noll, 1967). This renders our model physically motivated from the viewpoint of inhomogeneous plastic deformations at small scales.

We apply our theory to study the plastic behavior of a semi-infinite thin slab, under tensile loading, having a planar incoherent interface (see Fig. 2). The incoherency of the interface is essentially characterized by a non-trivial jump in plastic strain across the interface. In fact, for the thin slab problem, the driving force for interface propagation comes out to be proportional to this jump. As a result, interface is driven to motion within the slab only if it is incoherent. We consider two cases to illustrate our theory. First, we assume the interface to be located in the middle of the slab. The symmetry of the problem prevents incoherency and consequently any motion of the interface. The achieved simplicity however allows us to obtain an analytical solution, which is used to understand the nature of the two internal length scales as well as investigate the size effects on the overall plasticity of the slab. In the second case, we assume the interface to be located asymmetrically in the slab such that the amplitude of plastic strain in the thinner side is almost an order smaller than the other side. The interface becomes incoherent immediately after yielding and subsequently propagates into the thinner region. In both the cases, yielding in the bulk precedes that at the interface. This is reasonable since inhomogeneities (such as dislocations) pile up near the interface during bulk plastic flow, and start to rearrange abruptly at certain critical stress, leading to strain burst and eventual yielding at the interface (Morris et al., 2007; Chen and Gottstein, 1988). Once yielded, the interface encourages stress relaxation in the slab by allowing defects to transmit from one bulk region into the other. For instance, the dislocations, which get dissociated from the interface, get impinged into the adjacent bulk, and a fraction of the bulk dislocations get accumulated at the boundaries to modify their structural and mechanical behavior (cf. Chapter 12 of Sutton and Balluffi (2003)). Furthermore, if driven to move, the interface helps in further relaxation of stress by accommodating the cumulative strain in its neighborhood. Interestingly, when the interface propagates with a finite speed, the effective plastic behavior of the slab exhibits both strain softening and strain-rate dependency. The softening is more prominent for highly mobile interfaces. On the other hand, increase in the average strain-rate raises the maximum stress attained by the slab, thereby delaying the softening response. These findings are in confirmation with the available experimental observations (Morris et al., 2007; Chen and Gottstein, 1988; Legros et al., 2008).

The earliest formulation to include strain gradients within a plasticity theory was proposed by Aifantis (1984; 1987) in an attempt to develop a microstructurally informed macroscopic theory of plastic flow. Departing from the classical theories, he modified the flow stress to include a linear dependence on Laplacian of the effective plastic strain. This led to introduction of an internal length scale in the theory and eventually to an effective prediction of size dependent material response. The strain-gradient effect was alternatively introduced by Fleck et al. (1994) and Fleck and Hutchinson (1997) using the framework of couple stress theory. In a significant development, Fleck and Hutchinson (2001) used principle of virtual power to derive a broad class of strain-gradient plasticity models, where the notion of effective plastic strain was also generalized so as to introduce multiple internal length scales within the same framework. The theory was however found to be inconsistent with the laws of thermodynamics (Gurtin and Anand, 2009; Gudmundson, 2004). A thermodynamically consistent strain-gradient theory was developed, again using the principle of virtual power but incorporating a strain-gradient dependent defect energy, by Gurtin and Anand (2009). Our strain-gradient plasticity model is fundamentally different to these in that we include a dependence on second gradient of strain (through the incompatibility tensor) constitutively in the yield criteria. We do not consider contributions from defect energy, neither do we use the principle of virtual power.

The strain-gradient plasticity models for bulk deformation were extended to include interface energy in the works of Aifantis and Willis (2004; 2005; 2006). It is in these works that we find the first discussions of interfacial versus bulk yielding (Aifantis and Willis, 2005), ‘knee’ like feature in stress-strain relations (Aifantis and Willis, 2004), and the associated phenomena of strain burst captured through nanoindentation experiments (Aifantis et al., 2006; Aifantis and Willis, 2005). A detailed comparison between gradient plasticity models with experimental data illustrating a ‘knee’ is also available in their more recent works (Zhang et al., 2014a; Zhang and Aifantis, 2015). There have been several other proposals and applications of strain-gradient plasticity theories which include interfacial effects (Basak and Gupta, 2016; Al-Rub, 2008; Fleck and Willis, 2009; Fredriksson and Gudmundson, 2007; 2005; Polizzotto, 2009; Gudmundson, 2004; Gupta and Steigmann, 2012; Pardoen and Massart, 2012; Voyiadjis et al., 2014; Wulfinghoff et al., 2015; 2013). Our interfacial plasticity model is distinct from these in the way strain gradients have been incorporated in the interface yield criteria. Our dependence is motivated from the incompatibility of plastic strain at the interface. Furthermore, ours is the first strain-gradient plastic model, to the best of our knowledge, which includes mobile interfaces such as those present during phase/grain boundary propagation. In this way, we are able to study the effect of dynamic interfaces on the overall strain-gradient plasticity of the solid.

We have organized the paper as follows. A general theory of small deformation plastic flow in isotropic solids with a propagating planar incoherent interface has been developed in Section 2. It includes formulating the necessary kinematics, deriving local dissipation inequalities, and consequently proposing plastic flow rules and interfacial kinetics. In Section 3, the general theory is simplified towards posing an initial-boundary value problem for investigating the plastic deformation of a semi-infinite thin slab, as shown in Fig. 2. The problem is solved, and the results are discussed in detail, in Section 4 first for a stationary interface within