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Three-dimensional poroelastic effects during hydraulic fracturing in permeable rocks

Saeed Salimzadeh[a,b,*], Adriana Paluszny[a], Robert W. Zimmerman[a]

[a] Department of Earth Science and Engineering, Imperial College, London, UK
[b] Centre for Oil and Gas, Technical University of Denmark, Lyngby, Denmark

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ABSTRACT

A fully coupled three-dimensional finite-element model for hydraulic fractures in permeable rocks is presented, and used to investigate the ranges of applicability of the classical analytical solutions that are known to be valid in limiting cases. This model simultaneously accounts for fluid flow within the fracture and rock matrix, poroelastic deformation, propagation of the fractures, and fluid leakage into the rock formation. The model is validated against available asymptotic analytical solutions for penny-shaped fractures, in the viscosity-dominated, toughness-dominated, storage-dominated, and leakoff-dominated regimes. However, for intermediate regimes, these analytical solutions cannot be used to predict the key hydraulic fracturing variables, *i.e.* injection pressure, fracture aperture, and length. For leakoff-dominated cases in permeable rocks, the asymptotic solutions fail to accurately predict the lower-bound for fracture radius and apertures, and the upper-bound for fracture pressure. This is due to the poroelastic effects in the dilated rock matrix, as well as due to the multi-dimensional flow within matrix, which in many simulation codes is idealised as being one-dimensional, normal to the fracture plane.

1. Introduction

Hydraulic fracturing is the process by which one or more fractures are propagated into a rock formation, driven by the internal flow of a pressurised fluid. While fluid-driven fracturing can occur naturally, it is most often studied within the context of the engineering process of injecting fracturing fluid into a reservoir rock, with the aim of increasing well productivity (Adachi et al., 2007; Bazant et al., 2014). Although the hydraulic fracturing process is currently often thought of in the context of shale gas reservoirs, in current industry practice, almost all oil and gas wells are hydraulically fractured (Economides and Nolte, 2000). Hydraulic fracturing is a complex, multi-physics, multi-dimensional problem, which requires robust models that can simultaneously account for matrix and fracture deformation, fluid flow through the matrix and fractures, fluid exchange between fractures and matrix, and fracture propagation and interaction, all in a fully-coupled, three-dimensional setting.

Hydraulic fracturing protocols are designed to control the fracture's surface area and aperture distribution, and also aim to control injection pressure, and the dependence of these variables on fracturing fluid rheology, injection rate, and the hydro-mechanical properties of the rock (Detournay and Peirce, 2014). Analytical and semi-analytical solutions have been developed to quantify hydraulic fracturing variables of interest, such as injection pressure, fracture aperture, and fracture length (*cf.* Adachi et al., 2007). These solutions provide the foundation for hydraulic fracturing design (*e.g.* Cleary, 1980; Cleary et al., 1988). These solutions are constructed by combining the equations for laminar flow through the fracture, with the equations for elastic deformation of the adjacent rock. Fluid flow through the fracture is commonly modelled using lubrication theory, which is derived from the general Navier–Stokes equation for flow of a fluid between two parallel plates (Batchelor, 1967; Zimmerman and Bodvarsson, 1996), whereas the fracture aperture is calculated using linear elasticity in conjunction with Linear Elastic Fracture Mechanics (LEFM) to compute the mode I stress intensity factor at the fracture tip (Geertsma and de Klerk, 1969; Spence and Sharp, 1985).

Based on the energy-dissipation mechanism, fracture propagation regimes can be classified as *viscosity-dominated* or *toughness-dominated* (Detournay, 2004). In the viscosity-dominated regime, energy dissipation is dominated by the flow of the viscous fluid, whereas in the toughness-dominated regime, energy dissipated is dominated by the creation of new fracture surfaces at the fracture

* Corresponding author at: Centre for Oil and Gas, Technical University of Denmark, Lyngby, Denmark.
E-mail address: saeeds@dtu.dk (S. Salimzadeh).

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tip. Based on the ability of the rock matrix to dissipate fracturing fluid, two other extremes can be defined: *storage-dominated*, in which the injected fluid remains mainly inside the fracture, and *leakoff-dominated*, in which most of the injected fluid dissipates into the surrounding medium. The four resulting combined asymptotic regimes are therefore *storage-viscosity, storage-toughness, leakoff-viscosity*, and *leakoff-toughness* (Garagash et al., 2011). Asymptotic solutions that are valid at the end-members of the parameter space provide a fundamental understanding of the hydraulic fracturing process, and provide benchmarking cornerstones for numerical models. However, existing analytical solutions are restricted to simplified fracture geometries in homogeneous rock masses, and are typically constrained to a set of fixed boundary conditions. Standard geometries include the PKN fracture (Perkins and Kern, 1961; Nordgren, 1972; Mathias and van Reeuwijk, 2009), the KGD fracture (Geertsma and de Klerk, 1969; Spence and Sharp, 1985; Adachi and Detournay, 2008), and radial (penny-shaped) fractures (Savitski and Detournay, 2002; Bunger et al., 2005; Kovalyshen, 2010). Moreover, analytical solutions do not exist for cases that are not at the corners of this parameter space.

Numerical models that attempt to simulate hydraulic fracturing include the boundary integral method (Peirce and Siebrits, 2001), the boundary element method (Simpson and Trevelyan, 2011), the distinct element method (Marina et al., 2014), the finite element method (Carrier and Granet, 2012), discrete fracture network (Fu et al., 2013), the embedded fracture model (Norbeck et al., 2015), the lattice approach (Grassl et al., 2015) and the extended finite element method (Dahi-Taleghani, 2009; Mohammadnejad and Khoei, 2013; Salimzadeh and Khalili, 2015a). However, in the majority of available models, flow through the rock matrix, and fluid exchange between fracture and rock matrix, are either ignored by assuming an impermeable rock formation (*e.g.* Dahi-Taleghani, 2009), or simplified by using a one-dimensional analytical leakoff model (*e.g.* Zhou et al., 2015). Substantial field evidence has proven the impermeable matrix assumption to be an unrealistic assumption (Economides and Nolte, 2000; Adachi et al., 2007). In one-dimensional leakoff models (Carter, 1957), fracture-to-matrix flow is represented as a sink term in the mass balance equation for fracture flow. This approach has several shortcomings, such as the assumption of one-dimensional flow, time-dependency of flow instead of pressure-dependency, and more importantly, this approach cannot model matrix dilation. Although flow from the fracture into the rock matrix is by definition locally one-dimensional at the fracture wall, where the flux vector must be normal to the fracture wall, in a global sense it is three-dimensional, unless the permeability in the direction normal to the fracture plane is significantly higher than in other directions (Hagoort et al., 1980). As time elapses, the leakoff rate predicted by Carter's model, at each position along the fracture, decreases proportionally to square-root of time; consequently, a scenario of fracture arrest is not possible (Mathias and van Reeuwijk, 2009). Finally, this model does not account for the fact that seepage of the fracturing fluid into the rock formation increases the fluid pressure in the matrix, causing dilation of the rock matrix. A dilated matrix applies stresses back onto the fracture, referred to as 'back-stresses' in the hydraulic fracturing literature, which tend to close the fracture (Kovalyshen, 2010). These factors also affect the available semi-analytical solutions for leakoff-dominated regimes that use a simplified one-dimensional leakoff model in their formulation. These solutions therefore fail to accurately predict hydraulic fracturing parameters in leakoff-dominated regimes, as shown by Carrier and Granet (2012), and Salimzadeh and Khalili (2015a) for single-phase flow, and by Salimzadeh and Khalili (2015b) for two-phase flow in two dimensions, as well as in the present study for three dimensions.

In addition to poroelastic effects due to the aforementioned back stress phenomenon, there is a further environmental consequence of fluid seepage through the rock matrix, as it may promote the possible migration of injected fluid towards drinking water aquifers (Birdsell et al., 2015). Therefore, robust modelling of matrix flow is essential for both hydraulic fracture engineering and environmental aspects of subsurface fracturing. Only a few attempts have been made to incorporate flow in the rock matrix, coupled to mechanical deformation and flow in fracture. Rethore et al. (2008), Mohammadnejad and Khoei (2013) and Khoei et al. (2014), using the extended finite element method, introduced enriched pressures at the fracture to capture the discontinuous flow velocity at the fracture boundary. However, the enriched pressure represents the fluid pressure in the rock matrix *near* the fracture, and does not represent the pressure *inside* the fracture. Therefore, when coupled with mechanical deformation, the enriched pressure will be scaled by the Biot coefficient, whereas the fracture pressure actually does not require such scaling. Carrier and Granet (2012) introduced independent flow through the fracture and the rock matrix into their hydraulic fracture model. Their model was a combination of zero-thickness elements for the propagating fracture, and conventional bulk finite elements with a cohesive zone model. The equality of pressure between fracture and matrix at the fracture walls was enforced in the numerical model using Lagrange multipliers. Salimzadeh and Khalili (2015a, b) proposed an XFEM model that included two independent flow models in the fracture and the rock matrix, with a leakoff mass transfer between fracture and rock matrix to link the two. The leakoff depends on the pressure gradient in the matrix adjacent to the fracture, as well as on the fluid viscosity and matrix permeability. Norbeck et al. (2015), using an embedded fracture model, also considered two flow domains for matrix and fracture in two dimensions, and linked them through a similar mass transfer term.

A three-dimensional fully coupled finite element model for hydraulic fracturing is presented in the present paper, validated against known analytical solutions, and subsequently applied to study the influence of fluid exchange between fracture and matrix on fracturing. In particular, 3D diffusion and its related poroelastic effects on the propagation of fractures are investigated. The present model accounts for fluid flow within fracture and matrix, the propagation of the fracture, and fluid leakage into the formation rock. Fluid flow through the permeable rock matrix is modelled using Darcy's law, and is coupled with laminar flow within the fracture. Fracture growth and the direction of growth are estimated using an energy-based criterion that is based on the modal stress intensity factors along the fracture tip (Paluszny and Zimmerman, 2013). This model is validated against available asymptotic solutions for penny-shaped hydraulic fractures. Fifteen cases with varying fluid and rock matrix properties are run, to investigate the impact of fluid and rock matrix properties on the leakoff and fracturing. Numerical simulations conducted over a range of parameter values delineate the limits of validity of the various available asymptotic solutions.

2. Computational model

Fractures are represented discretely using two-dimensional surfaces embedded in a three-dimensional domain. When deriving the governing equations, each fracture is represented by a discontinuity Γ_c in the domain Ω with boundary Γ, as shown in Fig. 1. The fully coupled model is constructed on three separate yet interacting sub-models, including models for mechanical deformation, fracture flow, and matrix flow. The solid matrix is assumed to be linear elastic, homogeneous, and isotropic, with flow modelled using Darcy's law. An independent fracture flow model is developed based on lubrication theory. The mechanical and fracture