International Journal of Solids and Structures 000 (2016) 1–18

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Effective anisotropic compliance relationships for wing-cracked brittle materials under compression

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ARTICLE INFO

Article history:
Received 15 April 2015
Revised 22 March 2016
Available online xxx

Keywords:
Compressive compliance of damaged brittle materials
Damage induced anisotropy
Wing-crack modeling
Periodic unit cell

ABSTRACT

The failure mechanism for brittle materials under compressive loading is generally assumed to be dominated by the development of wing-cracks. Due to the preferential direction of the wing-crack development, material damaged in this mode exhibits damage induced anisotropy and volume dilatancy. The effective compliance of such wing-crack damaged materials is important for developing micromechanics-based constitutive models. This work addresses the scenario of a brittle material with periodically distributed wing-cracks under uniaxial compressive load. Closed form expressions for the instantaneous anisotropic compliance tensor with respect to the key physical parameters of the wing-cracks (the number density of cracks, the pre-existing flaw orientations and flaw sizes, the instantaneous length of wing-cracks, and the friction coefficient on the flaw surfaces) are derived through a kinematic approach. Accordingly, finite element models with perturbing compressive loads and periodic boundary conditions are carried out based on the above ideological wing-crack model, in order to verify and parameterize the analytically-based model. Good agreement is found between the finite element results and the analytical expression.

1. Introduction

Brittle materials, such as ceramics, concrete and rock, exhibit highly non-linear and complex overall response to external loads before total failure, such as anisotropy, dilatation, hysteresis and strain rate hardening phenomena. Such material behaviours are associated with the damage process of crack growth and coalescence. In addition to the influence of load path, the crack growth processes are profoundly related to the pre-existing flaws on the micro-scale, which include weak grain boundaries, microcracks, triple junctions, second phases, pores, soft or hard inclusions, etc. The population of such flaws can be characterized by number density, distributions of size and orientation, aspect ratios, clustering and so forth. Although the material typically exhibits isotropic macro-scale properties in the virgin state due to the randomly oriented pre-existing flaws, the preferential crack growth directions under applied loads causes the constitutive descriptions of the solid to become highly anisotropic.

Due to the contact at some flaw-crack interfaces, it is particularly challenging to characterize the mechanical response of damaged brittle materials under compressive loads. Numerous experimental studies have reported that under uniaxial compressive loads without confinement, specimens fail by axial cracking (or splitting) accompanied by significant inelastic volume increment and dilatation transverse to the loading direction (see for example, Brace and Bombolakis, 1963; Brace et al., 1966; Peng and Johnson, 1972; Zoback and Byerlee, 1975). These studies also observed that lateral confinement strongly affects the inelastic behaviour and enhances the material strength. Brace and Bombolakis (1963) proposed a frictional sliding model which connects the observed phenomena of axial cracking and dilatation with the pre-existing flaws, generally referred to as wing-cracking (see Fig. 1a). Under uni-axial compressive loads, pre-existing flaws are closed and the material tends to slide on the flaw surfaces that are inclined relative to the primary loads. This sliding increases the stress intensity at the tips/edges of the pre-existing flaws and opens new crack surfaces when the critical stress intensity factor is reached. The path of each wing-crack follows the maximum energy release rate, and aligns with the maximum compressive loading direction once it is fully developed (i.e., in the x_1 direction in Fig. 1a, assuming a compressive stress σ_{11}). Ultimately, failure in an axial splitting mode is considered to be the consequence of the growth and coalescence of such wing-cracks.

Different aspects of the frictional sliding model have been heavily investigated by authors including Moss and Gupta (1982), Nemat-Nasser and Horii (1982), Horii and Nemat-Nasser (1985),

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http://dx.doi.org/10.1016/j.ijsolstr.2016.08.012
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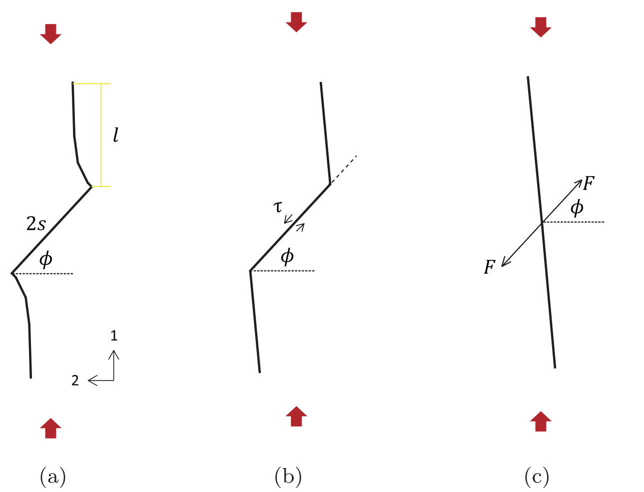

Fig. 1. Schematic of a typical wing-crack model. (a) Profile of a wing-crack of length l developed from a pre-existing flaw of size $2s$, showing the wing-crack re-alignment to the compressive loading direction as the crack grows. The contour of the re-alignment is plotted based on data from Nemat-Nasser and Horii (1982). (b) Simplified representation, ignoring the realignment process and assuming the wing-cracks to be straight; the average resolved shear stress at the flaw interface is noted as τ. (c) Further simplified by assuming a straight crack model with equal and opposite concentrated forces $F = 2s\tau$ applied at the center.

Horii and Nemat-Nasser (1986), Ashby and Hallam (1986), Nemat-Nasser and Obata (1988), Deng and Nemat-Nasser (1992), Nemat-Nasser and Deng (1994), Basista and Gross (1998) and many others. In particular, Nemat-Nasser and Horii (1982) derived the analytical expression of the stress intensity factors at the tips of the wing-cracks and solved for the orientation of incremental crack growth; Deng and Nemat-Nasser (1992), Nemat-Nasser and Deng (1994) investigated the impact of the strain rate on the evolution of damage and the interaction between wing-cracks.

In recent years numerical models incorporating the wing-crack damage mechanism have been developed to simulate the failure of brittle materials under dynamic compressive loads. Compared to more phenomenological models describing the failure of brittle materials (e.g., the Johnson Holmquist model by Holmquist et al. (2001); Johnson and Holmquist (1999) or the more recent Kayenta model by Brannon et al. (2009)), the mechanics-based models provide a more direct connection to the underlying microstructure and its effect on the damage process. Deshpande and Evans (2008) and their subsequent work Deshpande et al. (2011) characterized three principal inelastic mechanisms that control the constitutive properties of brittle materials under the damage process: (a) plasticity due to lattice dislocation or twinning, (b) nucleation and development of micro-cracks, and (c) granular plasticity when the cracks are heavily coalesced with each other (saturated). Similarly, Paliwal and Ramesh (2008) developed a model implementing a self-consistent scheme, which was capable of capturing the influence of the statistics of pre-existing flaws and the applied strain rate on the stress-strain response. Hu et al. (2015) expanded this to a three-dimensional numerical model, and Tonge (2014) applied this constitutive model in the context of the Material Point Method (MPM). For all of the above-mentioned models, the instantaneous compliance with respect to damage is required. In the absence of a rigorous closed form solution for wing-crack damaged solids, effective material properties derived from tensile solutions, such as the isotropic form from Budiansky and O'Connell (1976) or the anisotropic form from Grechka and Kachanov (2006), have been applied in these simulations. These effective properties generally overestimate the compliance in the compressive loading direction, while underestimating the dilatation at a given damage state; therefore, they are not well suited for describing the mechanical response of a damaged material under compressive loads.

Walsh (1965) provided early studies of the effect of pre-existing flaws on the elastic modulus and Poisson's ratio of the material under compressive loads. Basista and Gross (1998); Kachanov (1982a, 1982b)); Moss and Gupta (1982); Nemat-Nasser and Obata (1988) analyzed the compressive behaviour of solids undergoing the wing-crack damage mechanism, and each developed expressions for the constitutive relations during the wing-crack development process. Results obtained from these analyses typically describe the dynamic relations between crack growth and material response, but they do not provide a closed form expression for the compliance in terms of the instantaneous damage state. Therefore, these solutions are not easily implemented in the micromechanical model. Furthermore, comparing with experimental results, the damage-induced dilatation predicted by these models is generally underestimated.

In the current paper the instantaneous two-dimensional compliance tensor is investigated, as a function of the characteristics of pre-existing flaws (density, sizes, orientations) and the associated wing-crack lengths. In Section 2 the general solution of a cracked solid is reviewed, and the specific solution for the wing-crack problem is derived through a kinematic relationship. In Section 3, the corresponding finite element simulations are presented to validate the derivation and to provide a fit to the analytical function defined in Section 2. Simplified expressions for the compliance tensor are introduced in Section 4, and a solution for more accurate wing-crack geometries is discussed in Section 5. Finally, a comparison between our solution and an earlier version developed by Nemat-Nasser and Obata (1988) is performed in Section 6.

2. Analytical approach

Under uniaxial compressive loads, wing-cracks are developed on the tips of pre-existing flaws due to the frictional sliding on the flaw faces. The growth direction of a wing-crack is controlled by the maximum energy release rate (see for example, Nemat-Nasser and Horii (1982)). As shown in Fig. 1a, a wing-crack starts at about 70° to the flaw surface, and then gradually realigns with the principal compressive loading direction as it grows. We call this the "realignment stage". For ease of analysis, the realignment stage is usually ignored and the wing-cracks are simplified as straight line cracks aligned at an angle towards the principal loading direction, see Fig. 1b. Some authors, e.g. Nemat-Nasser and Obata (1988) and Nemat-Nasser and Deng (1994), further simplified the representation to a straight crack, replacing the pre-existing flaw by forces applied in the center, as shown in Fig. 1c. In this case, the magnitude of the force is the flaw size multiplied by the average resolved shear stress at the flaw surface.

The analysis in this work is based on the simplified model with a straight wing-crack shown in Fig. 1b. Section 5 will address this simplification by considering wing crack realignment. Section 6 will show that the simplified straight crack model in Fig. 1c may not provide a good approximation for the material response, in particular when the wing-cracks do not realign along the center of the flaw.

2.1. General solution for cracked solid

Our solution is limited to two dimensions, although extension to a three-dimensional solution may be possible using appropriate dimensionality laws. The macroscopic average strain and stress tensor are denoted as $\bar{\boldsymbol{\epsilon}}$ and $\bar{\boldsymbol{\sigma}}$, which consist of the components:

$$\bar{\boldsymbol{\epsilon}} = \begin{bmatrix} \bar{\epsilon}_{11} & \bar{\epsilon}_{12} \\ \bar{\epsilon}_{21} & \bar{\epsilon}_{22} \end{bmatrix}, \quad \bar{\boldsymbol{\sigma}} = \begin{bmatrix} \bar{\sigma}_{11} & \bar{\sigma}_{12} \\ \bar{\sigma}_{21} & \bar{\sigma}_{22} \end{bmatrix}. \tag{1}$$