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On the dispersion of the axisymmetric longitudinal wave propagating in a bi-layered hollow cylinder made of viscoelastic materials

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ABSTRACT

The paper deals with the study of axisymmetric longitudinal fundamental wave propagation (dispersion) in the bi-layered hollow circular cylinder made of linear viscoelastic materials. The investigations are made by utilizing the exact equations of linear viscoelasto-dynamics. The corresponding dispersion equation is derived for an arbitrary type of hereditary operator and the algorithm is developed for its numerical solution. Concrete numerical investigations are made for the case where constitutive relations of the layers' materials of the cylinder are described through fractional exponential operators. The influence of the viscosity of the layers' materials of the hollow cylinder on the wave dispersion is studied through the rheological parameters which indicate the characteristic creep time and long-term values of the elastic constants of these materials. Dispersion curves are presented for certain selected dispersive and non-dispersive attenuation cases under various values of the problem parameters and the influence of the aforementioned rheological parameters on these curves is discussed. As a result of the numerical investigations, in particular, it is established that in the case where the rheological parameters of the layers are the same, the viscosity of the layers' materials causes the axisymmetric wave propagation velocity to decrease.

1. Introduction

It is known that the study of time-harmonic wave dispersion and attenuation in viscoelastic materials and in elements of constructions made from these materials has great significance not only in the fundamental (theoretical) sense, but also in the application sense. As examples of such applications are the nondestructive inspection of tubes and pipes which are used in the infrastructure of many industries such as gas, oil, and water transport. In many cases these tubes are coated with viscoelastic polymer coatings for corrosion protection and therefore, under nondestructive testing of the tubes with guided waves, it is necessary to know the attenuation and dispersion rules of the waves propagating therein.

Another example of the application of the study of the propagation of guided waves in viscoelastic materials and constructions made of viscoelastic elements is the use of viscoelastic systems for attenuation of vibrations and waves caused by an earthquake or with various types of sound sources. These and many other application fields from the results of studies of wave propagation in viscoelastic bodies necessitates investigation of related problems from both the theoretical and experimental aspects. Nevertheless, up to now, investigations related to wave propagation in structural elements made from viscoelastic materials have not been as numerous as studies which have been made for the same structural elements made from purely elastic materials. We attempt to give a brief review of these investigations and begin with the papers by Weiss (1959) and Tamm and Weiss (1961) which relate to the Lamb wave propagation in an isotropic viscoelastic layer with stress-free surfaces. In these papers, it is assumed that the elastic constants are complex and independent of frequency. Coquin (1964) proposed an approximate method for investigation of the Lamb wave propagation in a plate from viscoelastic materials with small losses and frequency-dependent elastic moduli. The influence of low-compressibility materials with real Poisson's ratio and frequency dependent complex shear moduli on the propagation of Lamb waves was investigated by Chervinko and Shevchenko (1986).

Simonetti (2004) also studied Lamb wave propagation in elastic plates coated with viscoelastic materials and analyzed the effect of damped coatings on the dispersion characteristics of waves

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in these plates. The results reviewed above were noted and/or detailed in the monograph by Rose (2004).

In a paper by Wolosewick and Raynor (1967) axisymmetric non-stationary torsional wave propagation in the semi-infinite circular cylinder was considered for the case where an arbitrary radial axially symmetric tangential shear stress distribution harmonic in time acts on the end of the cylinder.

In a paper by Barshinger and Rose (2004) axisymmetric longitudinal guided wave dispersion and attenuation in a metal elastic hollow cylinder coated with a polymer viscoelastic layer was studied. The viscoelasticity of the coated layer was taken into consideration through attenuation coefficients of the longitudinal and shear waves in the corresponding viscoelastic materials. These coefficients are determined experimentally for the frequencies in the order 1–5 MHz and are used for determination of the corresponding complex moduli. Consequently, using these complex moduli the wave dispersion and attenuation dispersion in the bi-layered hollow cylinder was investigated.

Note that the approaches based on the frequency independent complex modulus are useful for determination of the instantaneous influence of the polymer coatings on the wave attenuation and wave propagation velocity for the certain narrow interval of the wave frequencies under which the imaginary part of the complex modulus is determined experimentally. Consequently the theoretical results obtained in the frequency independent complex modulus cases can be employed in the foregoing sense under non-destructive evaluation of the elements of construction such as the steel-polymer bi-layered hollow cylinder which was made in the aforementioned paper by Barshinger and Rose (2004).

There are also investigations, such as made by Kirby et al. (2013), in which the influence of the viscoelasticity of the coatings of the pipes on the scattering of the longitudinal and torsional waves. Note that in this paper the viscoelasticity of the coating is modeled as in the paper by Barshinger and Rose (2004).

In the paper by Leinov et al. (2015) the attenuation of the torsional and longitudinal waves propagation in pipe buried in sand is investigated experimentally. This attenuation appears as a result of the energy leakage into the embedding soil.

The paper by Jiangong (2011) deals with the study of the viscoelastic SH waves in functionally graded material and laminated plates. The Kelvin–Voigt viscoelastic model is used for describing rheological relations of the materials. It is established that the viscous effect on dispersion curves and the attenuation are determined not only by the viscous coefficients but also by the elastic constants.

In the paper by Manconi and Sorokin (2013) the classical Rayleigh–Lamb problem for a viscoelastic plate is studied. The non-dispersive attenuation case is considered and numerical results are presented and discussed for illustration the role of dispersion in the cut-off phenomenon and in the phenomenon of veering for dispersion curves. The viscoelasticity model employed in the paper by Manconi and Sorokin (2013) was also used in the papers by Bartoli et al. (2006), Mace and Manconi (2008), Manconi and Mace (2009), and Mazotti et al. (2012). Note that in these papers, the viscoelasticity of the materials is modelled by the well-known Kelvin–Voigt model. At the same time, in these works the hysteretic model is also employed.

The foregoing investigations can also be classified according to the numerical solution method. In connection with this it can be noted that investigations carried out in the papers by Bartoli et al. (2006), Mazotti et al. (2012) and others listed therein have been made by employing the semi-analytical finite element (SAFE) method. The SAFE method is based on the presentation of the sought values and multiplying $\exp i(kz - \omega t)$. According to this presentation and after doing some obvious mathematical manipulations, the 3D problems (2D problems) are reduced to the 2D problems (1D problems) under the corresponding finite element modeling.

Moreover, in the papers by Mace and Manconi (2008), Manconi and Mace (2009), Manconi and Sorokin (2013) and others listed therein, the wave finite element method (WFE) is employed. The method is based on the presentation of the sought values with multiplying $\exp i\omega t$ and uses the periodicity relations between the nodal displacements in the wave propagation direction. As a result of this, the segment matrices are projected onto the degree of freedoms of some selected node or nodes. Note that under employing the WFE before the use of the aforementioned periodicity conditions, the corresponding stiffness and mass matrices are performed by employing the conventional FE.

We also note the paper by Hernando Quintanilla et al. (2015) which relates to the study of the guided wave dispersion in an anisotropic viscoelastic layered medium, the viscoelasticity of which is also modeled through the Kelvin–Voigt model. However, in this paper the corresponding eigenvalue problem is solved by employing the spectral collocation method (SCM) which also uses the presentation $\exp i(kz - \omega t)$ of the sought values and has some advantages with respect to other numerical solution methods. Note that SCM is based on the replacement of the partial differential operators matrix with the corresponding finite difference matrix under which the non-uniform Chebyshev grid of N points is used.

In the papers by Meral et al. (2009, 2010), the fractional order Voigt (or Kelvin–Voigt) model is used for investigation of 2D dynamic problems for viscoelastic materials. In these papers, the ordinary derivative $\partial/\partial t$, with respect to time in the classical Voigt model (which is also used in all the above reviewed papers) is replaced with the fractional order derivative $\partial^\alpha/\partial t^\alpha$ in the Weyl sense. Consequently, according to this replacement, the new rheological parameter α is introduced and this parameter allows the viscoelasticity properties of the material to be described more accurately than the classical Voigt model. For instance, in the paper by Meral et al. (2010), using the fractional order Voigt model, Lamb wave propagation and attenuation were studied theoretically and verified experimentally for a tissue mimicking phantom material. In this paper, it was established that successful selection of the rheological parameter α allows for improvement in matching theory to experiment. Note that the aforementioned fractional order Voigt model is also successfully employed for describing the time-dependent mechanical behavior of the biological tissue which is also indicated in the papers by Meral et al. (2009, 2010) and other ones listed therein.

With these we restrict ourselves to consideration of the review of the related studies from which follows that the investigations on the dispersion of guided waves in the plates or cylinders made from viscoelastic materials were carried out mainly in the following cases:

- The complex modulus of viscoelastic materials is taken as frequency independent (the hysteretic model);
- The viscoelasticity of the materials is described by the simplest models such as the classical Kelvin–Voigt or simplest fractional Kelvin–Voigt models; and
- The expression for the complex elasticity modulus is obtained experimentally for concrete polymer materials.

Consequently, in the works considered above, the corresponding investigations on wave dispersion and attenuation were not connected with the more complicated and real models for viscoelastic materials and the numerical results obtained in these works do not illustrate the character of the influence of the rheological parameters of the viscoelastic materials on this dispersion. To address this, the first attempt was made in a paper by Akbarov and Kepceler (2015) in which the torsional wave dispersion in the sandwich hollow cylinder made from linear viscoelastic materials,