International Journal of Solids and Structures 000 (2016) 1–12

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Effect of a curved fiber on the overall material stiffness

Borys Drach*, Dmytro Kuksenko, Igor Sevostianov

Department of Mechanical and Aerospace Engineering, New Mexico State University, Las Cruces, NM, USA

A R T I C L E I N F O

Article history:
Received 2 March 2016
Revised 15 August 2016
Available online xxx

Keywords:
Micromechanics
Stiffness contribution tensor
N-tensor
Curvilinear fiber
Curved tow
Replacement relations

A B S T R A C T

The effect of fiber curvature on the overall material stiffness of fiber-reinforced composites is discussed using the formalism of stiffness contribution tensor for the special case of a continuous sinusoidal fiber. Stiffness contribution tensors of individual sinusoidal fibers with different crimp ratios are presented for the first time. The tensors are calculated numerically using Finite Element Analysis and analytically by representing fibers as equivalent sets of ellipsoids following the procedure available in the literature for approximation of effective composite stiffness. It is demonstrated that the existing procedure results in large approximation errors in several components of the stiffness contribution tensors of the considered fibers. A modification to the procedure is proposed to improve the accuracy of the predictions. Replacement relations that interrelate stiffness contribution tensors of inhomogeneities having the same shape but different material properties are tested for the studied fiber geometries.

1. Introduction

This paper focuses on the quantitative evaluation of the effect produced by an isolated curved fiber of circular cross-section on the overall elastic properties. We use the concept of stiffness contribution tensor of an individual inhomogeneity (see, for example, Sevostianov and Kachanov (2002a); Kachanov et al. (2003); Drach et al. (2011)). Components of this tensor are calculated numerically and approximated by simple analytical formulas that allow identification of the microstructural parameters governing the aforementioned effect. The problem is motivated mostly by the needs of composite materials industry where the use of curved fibers has increased during the last two decades – in woven composites, composites reinforced with nanofibers, etc. Another important application is related to biomechanics of soft and hard tissues where collagen fibers usually have curved shape.

Effect of curvature on the fiber's mechanical response has attracted attention of researchers starting from 1950s. To the best of our knowledge, the first paper where geometrical parameters of bent yarns were analyzed in the context of stress-strain analysis was the one of Backer (1952). The author derived expressions for local fiber tensile strain, average strain in the helix half loop, etc. in terms of the geometric quantities. The main application of the earlier research in this area (see also works of Platt (1950a); Platt (1950b); Schwarz (1951)) was textile materials and their elastic performance.

With appearance of fiber reinforced composites, the problem was transferred to the area of optimization of mechanical performance of composites. Bažant (1968) analyzed the influence of the curvature of reinforcing fibers on the mean longitudinal modulus and strength of composite materials. Brandmaier (1970) showed that the maximum composite strength can be obtained when most of the stress is carried in the fiber direction. It was first pointed out that the optimal fiber orientation is different from the principal stress direction depending on the strength properties of the lamina. Later research can be categorized into four main directions:

a) optimization of the properties of materials containing curvilinear fibers using numerical methods;
b) calculation of the effective stiffness of a single curvilinear fiber considered separately from the matrix material;
c) solution of boundary value problems for materials with curved fibers;
d) approximation of the effective properties of woven composites.

First works on numerical optimization appeared in 1980s. Hyer and Charette (1987) used Finite Element Analysis (FEA) and an iterative scheme to find optimal orientation of curved fibers in laminaes to increase the material strength. The authors also cite PhD dissertation of Cooper (1972) where the effect of fiber curvature was first analyzed in the context of mechanical properties. Gurdal and Olmedo (1993) obtained a solution to the plane elasticity problem for a symmetrically laminated composite panel with spatially varying fiber orientations and discussed the effects of the variable fiber orientation on the displacement fields, stress

* Corresponding author. Fax: +1 (575) 646 6111; Tel.: +1 (575) 646-8041.
E-mail addresses: borys@nmsu.edu (B. Drach), kuksenko@nmsu.edu (D. Kuksenko), igor@nmsu.edu (I. Sevostianov).

http://dx.doi.org/10.1016/j.ijsolstr.2016.08.018
0020-7683/

resultants, and global stiffness. Reuschel and Mattheck (1999) and Mattheck and Bethge (1998) discussed optimization of three-dimensional (3-D) fibrous structures mimicking natural structures such as bones and trees to design laminated composites which follow the principal stress direction selection scheme. Setoodeh and Gürdal (2003) used the method of Cellular Automata for curvilinear fiber design of composite laminae for in-plane loading. Duvaut et al. (2000) used variational formulation to develop a numerical method for determining the optimal direction and volume fraction of fibers at each point of a structure.

Effective stiffness of a curvilinear fiber without matrix was first addressed by Potierferry and Siad (1992) who applied the double-scale asymptotic expansion method to a corrugated beam when the ratio of the length of the basic cell to the length of the beam tends to zero. Ding and Selig (2004) computed the effective compliance of a helical spring by summing the compliances of infinitesimal elements along the path of the spring. Haussy et al. (2004) modelled a single yarn as an undulated beam and used perturbation method to calculate its effective stiffness. Messager and Cartraud (2008) computed axial stiffness of a helical beam-like structure using homogenization theory of periodic slender domains. Marino and Vairo (2012) developed a general model accounting for fiber 3D geometry, as well as for shear and torsional effects together with the extensional ones. They highlighted the influence of fiber geometric parameters and shear deformability, thus enabling to test the limits of applicability of the commonly used assumptions. Zhao et al. (2014) studied hierarchical helical structures by analyzing the internal forces and deformations of a single helical ply; the effect of hierarchical helical structures was revealed by comparing the properties of a carbon nanotube rope having two-level helical structure with its counterpart bundle consisting of straight carbon nanotubes. The authors examined dependence of mechanical properties of the materials on the initial helical angles, number of fibers, and handedness at different structural levels.

Exact solutions for elastic fields in materials containing thin curvilinear fibers loaded along the fiber axis have been obtained in the works of Akbarov and co-authors. In the book of Akbarov and Guz (2000) (Chapter 7) the basic principles of the solution are formulated. Kosker and Akbarov (2003) obtained the stress distribution in the body containing two neighboring periodically co-phase curved fibers. Akbarov et al. (2004); Akbarov et al. (2006) considered infinite elastic matrix containing a row of periodically co-phase and antiphase curved fibers, correspondingly.

The effective properties of composites with curvilinear fibers and tows can be estimated by considering the microstructure as an aggregate of infinitesimal “subcells” each having the properties of a composite reinforced by straight fibers aligned along the tow path. In the original approach, the subcells were assumed to be reinforced with infinitely long fibers and the overall elastic properties were calculated by averaging stiffness (upper bound) or compliance (lower bound) tensors of the subcells, see for example Kregers and Melbardis (1978); Gowayed and Pastore (1993); Gommers et al. (1996); Matveeva et al. (2014). Gommers et al. (1998) presented a Mori-Tanaka based method for homogenization of the subcells. It was shown in the aforementioned publications that use of straight infinitely long fibers for reinforcement in the subcells resulted in overestimation of the effective elastic properties of the composites. Huysmans et al. (1998) refined the approach by employing subcells reinforced by ellipsoids with finite aspect ratios determined by local tow curvature (also known as “short fiber analogy”) rather than infinite fibers. The latter approach was shown to produce relatively good predictions for textile composites especially for in-plane elastic properties (Birkefeld et al. (2012); Prodromou et al. (2011)) and it was later implemented in commercial software WiseTex (Verpoest and Lomov (2005); Lomov et al. (2007); Lomov et al. (2014)). A number of research groups have used the Mori–Tanaka based subcell approach since. For example, Mourid et al. (2013) applied it to homogenization of woven composites with viscoelastic properties, Skoček et al. (2008) used it to evaluate effective elastic properties of carbon/carbon composites, and Olave et al. (2012); Vanaerschot et al. (2013) utilized it to study the sensitivity of effective elastic properties of textile composites to variations in tow paths and cross-section geometries.

We note, however, that for anisotropic multiphase composites, Mori–Tanaka scheme may violate Hashin–Shtrikman bounds (Norris (1989) and Benveniste and Milton (2011)). Yet another inconsistency in the case of anisotropic multiphase composites is often claimed in the literature (Benveniste (1990); Qiu and Weng (1990); Qiu and Weng (1990)): the scheme predictions may violate the symmetry of the effective stiffness tensor. Artificial symmetrization does not resolve the problem (see Sevostianov and Kachanov (2014)). To the best of our knowledge, the results currently available in literature do not provide a tool that allows one to choose a homogenization scheme to evaluate the full set of anisotropic elastic constants of a material containing a given orientation distribution of curvilinear fibers.

This work addresses the lack of the tools mentioned above. Two approaches to calculation of the stiffness contribution tensors of individual continuous curvilinear fibers are presented for the first time: direct FEA and analytical approximation. Given elastic (compliance or stiffness) contribution tensor of a single inhomogeneity, effective elastic properties of a composite containing multiple inhomogeneities can be readily obtained using a number of homogenization schemes (e.g. Eroshkin and Tsukrov (2005); Chen et al. (2015); Drach et al. (2016)). Thus calculation of effective properties is outside of the scope of this paper. The presented analytical approximation for the *stiffness contribution tensor* is based on the approach proposed by Huysmans et al. (1998) for *effective stiffness tensor*. We demonstrate the applicability of the original approach to approximation of the stiffness contribution tensor of a curvilinear fiber and propose a modification to reduce the approximation error.

In the present study we focus on the stiffness contribution tensor of a single continuous sinusoidal fiber of circular cross-section embedded in a large matrix volume, see Fig. 1. The geometry of the fiber is characterized by amplitude a, wavelength λ and radius of the cross-section r. Amplitude and wavelength can be combined into a single parameter called “crimp ratio”, which is calculated as

$$CR = \frac{a}{\lambda} \tag{1.1}$$

Both matrix and fiber are assumed to have isotropic properties. We present the effects of the geometric parameters on the components of the stiffness contribution tensor.

We note that few results are available for description of the effect of non-ellipsoidal 3D inhomogeneities on the overall elastic properties. Compliance contribution tensors for several examples of pores of irregular shape typical for carbon/carbon composites were calculated by Drach et al. (2011) using FEA. In the narrower context of irregularly shaped cracks, certain results were obtained for compliance contribution tensors by Fabrikant (1989); Sevostianov and Kachanov (2002b) (planar cracks), Grechka et al. (2006) (intersecting planar cracks), Mear et al. (2007) (non-planar cracks), and Kachanov and Sevostianov (2012) (cracks growing from pores). Effects of various concave pores on overall elastic and conductive properties have been discussed in works of Sevostianov et al. (2008); Sevostianov and Giraud (2012); Chen et al. (2015) and Sevostianov et al. (2016). The results of the present research will contribute to the existing library of solutions for property contribution tensors of non-ellipsoidal inhomogeneities.

Please cite this article as: B. Drach et al., Effect of a curved fiber on the overall material stiffness, International Journal of Solids and Structures (2016), http://dx.doi.org/10.1016/j.ijsolstr.2016.08.018