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Partial slip contact of a rigid pin and a linear viscoelastic plate

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ABSTRACT

This paper analyzes partial slip contact problems in the theory of linear viscoelasticity under a wide variety of loading conditions, including cyclic (fretting) loads, using a semi-analytical method. Such problems arise in applications like metal-polymer contacts in orthopedic implants. By using viscoelastic analogues of Green's functions, the governing equations for viscoelastic partial-slip contact are formulated as a pair of coupled Singular Integral Equations (SIEs) for a conforming (pin-plate) geometry. The formulation is entirely in the time-domain, avoiding Laplace transforms. Both Coulomb and hysteretic effects are considered, and arbitrary load histories, including bidirectional pin loads and remote plate stresses, are allowed. Moreover, the contact patch is allowed to advance and recede with no restrictions. Viscoelasticity necessitates the application of the stick-zone boundary condition in convolved form, and also introduces additional convolved gap terms in the governing equations, which are not present in the elastic case. Transient as well as steady-state contact tractions are studied under monotonic ramp-hold, unload-reload, cyclic bidirectional (fretting) and remote plate loading for a three-element solid. The contact size, stick-zone size, indenter approach, Coulomb energy dissipation and surface hoop stresses are tracked during fretting.

Viscoelastic fretting contacts differ from their elastic counterparts in notable ways. While they shakedown just like their elastic counterparts, the number of cycles to attain shakedown states is strongly dependent on the ratio of the load cycle time to the relaxation time. Steady-state cyclic bulk hysteretic energy dissipation typically dominates the cyclic Coulomb dissipation, with a more pronounced difference at slower load cycling. However, despite this, it is essential to include Coulomb friction to obtain accurate contact stresses. Moreover, while viscoelastic steady-state tractions agree very well with the elastic tractions using the steady-state shear modulus in load-hold analyses, viscoelastic fretting tractions in shakedown differ considerably from their elastic counterparts. Additionally, an approximate elastic analysis misidentifies the edge of contact by as many as 7 degrees in fretting, showing the importance of viscoelastic contact analysis. The SIE method is not restricted to simple viscoelastic networks and is tested on a 12-element solid with very long time scales. In such cases, the material is effectively always in a transient state, with no steady edge-of-contact. This has implications for fretting crack nucleation.

1. Introduction

The contact of a viscoelastic polymer with a counterbody made of a much stiffer material has drawn considerable attention because of its importance in applications like rubber tires. There is a significant body of work focused on the frictional characteristics of soft elastomers like rubber (Carbone and Putignano, 2013; Grosch, 1963; Persson, 2001). Friction in such materials is attributed to bulk hysteretic losses, and typically investigated under conditions of rolling or sliding.

Early analytical work in viscoelastic contacts includes the solution of the Hertz-type problem with a monotonically increasing contact area (Lee and Radok, 1960), its extensions to non-monotonic cases (Graham, 1967; Hunter, 1960; Ting, 1966), and viscoelastic rolling contacts (Hunter, 1961; Morland, 1968). In plane-strain viscoelastic contacts, an extension of the Kolosov–Muskhelishvili method of linear elasticity has been used to solve both frictionless (Golden and Graham, 1988) and Coulomb limiting-friction problems (Goryacheva et al., 2008). More recent analytical / semi-analytical work has addressed such problems as adhesion in viscoelasticity (Hui et al., 1998) and the contact of viscoelastic bodies with hard, rough surfaces (Chen et al., 2011; Persson et al., 2004). However, the Coulomb partial slip regime in viscoelastic contacts has received much less attention. In partial slip contact, global relative tangential motion (sliding) does not occur. Instead, the contact consists of slip zones, where local relative

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tangential motion does occur, and stick zones, where no relative tangential motion occurs (Johnson, 1987).

Partial slip viscoelastic contacts arise in applications like orthopedic implants, which involve contact between a metal and a polymer (Duisabeau et al., 2004). The metal is typically austenitic stainless-steel or a titanium alloy like Ti-6Al-4V, while the polymer is usually Polymethyl Methacrylate (PMMA) or an Ultra-high Molecular Weight Polyethylene (UHMWPE) (Geringer et al., 2005). The failure of these implants is understood to be driven by a complex, contact-driven process known as fretting corrosion (Geringer et al., 2005; Tritschler et al., 1999), in which the deleterious effect of a corrosive environment enhances fretting (Higham et al., 1978). Cyclic loading of such contacts occurs during routine use of the limbs, e.g. while walking (Kim et al., 2013). Either the metal or the polymer may undergo damage, with wear debris from both materials reported in experiments (Tritschler et al., 1999). In metal-on-metal fretting, it is known that high-fidelity analysis of the edge-of-contact stress is an essential ingredient of fretting crack nucleation models (Fellows et al., 1995; McVeigh et al., 1999). This involves accurate modeling of partial slip contacts and tracking of the contact stress history during cyclic loading. Accurate contact stress analysis might be expected to provide similar insights into metal-polymer fretting.

The introduction of partial slip complicates contact problems in elasticity as well as in viscoelasticity. This is because of the history dependent nature of partial slip contacts, which makes their analysis inherently incremental, and coupling between pressure and shear tractions in the governing equations. An early partial slip solution in linear viscoelasticity was obtained by Goryacheva (1973) for rolling of a viscoelastic cylinder on a halfspace of a similarly viscoelastic material.[1] Other Coulomb frictional viscoelastic solutions include the rolling contact of layered cylinders (Kalker, 1991) and a cylinder rolling on a viscoelastic layer atop an elastic halfspace (Goryacheva and Sadeghi, 1995). The assumption of material similarity in Goryacheva (1973) or the Goodman approximation (Goodman, 1962) in Goryacheva and Sadeghi (1995) is analytically helpful because it eliminates coupling between the pressure and shear tractions. However, these assumptions are inapplicable to metal-polymer contacts, where the counterbody is much stiffer. Moreover, partial slip fretting contacts may be subject to very complex load histories. Under such conditions, it is almost impossible to obtain closed-form solutions. However, formulating the governing equations of contact as Singular Integral Equations (SIEs) leads to a fast, semi-analytical method to solve these problems.

The present work builds an accurate SIE-based model for partial slip viscoelastic contacts in monotonic and cyclic loading, thereby accounting for both Coulomb and hysteretic effects. The starting point is a viscoelastic analogue of the elastic Green's functions. Since metals are typically much stiffer than polymers, it is a good assumption to treat the metal as a rigid body.

The conforming (pin-plate) geometry is chosen for our work because it has the advantage of allowing calculation of indenter approach, and thus various energy dissipation estimates, in plane strain viscoelasticity. Furthermore, the implant contact geometry is typically of a conforming type. Conforming contacts also have the advantage of including halfspace contacts as a limiting case. Conforming elastic contacts have been studied extensively in both receding and advancing contact regimes (Gladwell, 1980; To and He, 2008). Two frictionless conforming contact solutions are known in linear viscoelasticity, both in the receding (rather than advancing) contact regime. Margetson and Morland (1970) considered the problem of separation of an inclusion from a viscoelastic plate in uniaxial loading. Subsequently, Golden and Graham (2001) considered the same problem with biaxial loading.

[1] This is the viscoelastic analogue of Carter's problem (Barber, 2010).

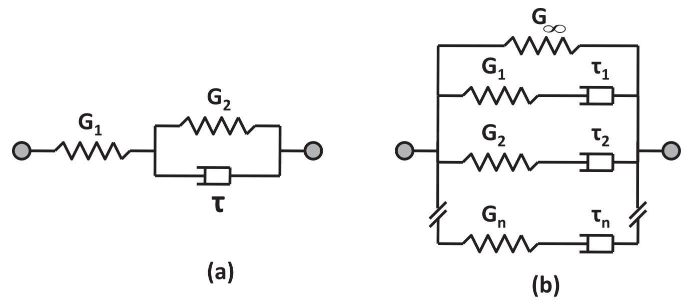

Fig. 1. (a) Three-element delayed elastic solid and (b) generalized Maxwell model.

2. Formulation

2.1. Creep and relaxation functions of a linear viscoelastic solid

The shear relaxation modulus of a linear viscoelastic solid is represented by (Golden and Graham, 1988) (Fig. 1)

$$\mathbf{G}(t) = G_\infty + \sum_{i=1}^{n} G_i \exp\left(-\frac{t}{\tau_i}\right) = G_0 - \sum_{i=1}^{n} G_i\left(1 - \exp\left(-\frac{t}{\tau_i}\right)\right) \tag{2.1}$$

where G_0 is the instantaneous modulus, G_i the shear moduli of the network spring elements, τ_r are relaxation time constants and G_∞ is the modulus at $t = \infty$. Similarly, the creep response is characterized by

$$\boldsymbol{J}(t) = J_\infty - \sum_{r=1}^{n} J_r \exp\left(-\frac{t}{\lambda_r}\right) = J_0 + \sum_{r=1}^{n} J_r\left(1 - \exp\left(-\frac{t}{\lambda_r}\right)\right) \tag{2.2}$$

where J_r and λ_r represent, respectively, the compliances of the springs and retardation time constants.

2.2. Governing equations of the contact problem

A general way to formulate contact problems for linear media is to use appropriate Green's functions. These are typically surface displacements produced by point normal and tangential loads acting on the boundary of the domain. We first derive a viscoelastic analogue of the elastic Green's functions for the plate, by using an extension of the Kolosov–Muskhelishivili formulation for viscoelastic materials (see Appendix).

Then, let $R_D < R$ be the radius of the rigid pin. In the reference state, the pin rests on the plate as shown in Fig. 2. If the pin is rotated by a small amount C_ω, and pressed into the viscoelastic plate by a rigid displacement $\overline{V} = (C_{0x}, -\Delta)$, the gap function $h_d(\theta, t)$ is

$$h_d(\theta, t) = (R - R_D)(1 + \sin(\theta)) - C_{0x}(t)\cos(\theta) + \Delta(t)\sin(\theta) \tag{2.3}$$

The overclosures thus produced must be relieved at every point by displacements in the viscoelastic plate so that the new gap equation is

$$h(\theta, t) + (R_D - R)(1 + \sin(\theta)) + C_{0x}\cos(\theta) - \Delta\sin(\theta) - \tilde{v}_r^\infty = \tilde{v}_r^p + \tilde{v}_r^q \tag{2.4}$$

Here $\tilde{v}_r^p$ and $\tilde{v}_r^q$ are, respectively, the radial surface displacements of the plate due to normal and shear tractions, and $\tilde{v}_r^\infty$ is the radial surface displacement caused by remote stresses applied to the