International Journal of Solids and Structures 000 (2016) 1–12

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Adaptive technique for discrete models of fracture

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ARTICLE INFO

Article history:
Received 3 May 2016
Revised 31 July 2016
Available online xxx

Keywords:
Discrete model
Random geometry
Statics
Adaptivity
Discretization
Concrete

ABSTRACT

Static discrete models are advantageously used for the simulation of fracture in quasibrittle heterogeneous materials. In order to correctly capture strain localization during the fracture process, it is often necessary to represent material heterogeneity in the model directly via its discrete geometry. Depending on the specimen size and the size of the heterogeneities, these simulations are typically extremely computationally demanding. The contribution aims to reduce this computational cost via the implementation of adaptivity in the construction of the discrete model geometry. The simulation starts with coarse discretization, which provides correct elastic behavior and is then adaptively refined during the simulation in regions that suffer high stresses that induce cracking and strain localization. The technique is applied in deterministic and probabilistic simulations and demonstrated on several examples.

1. Introduction

The discrete representation of materials is a natural alternative to continuous approaches. A collection of interconnected rigid bodies organized into a net structure is often called a discrete or lattice model. Discrete models are being used in current research for several purposes. Besides simulations of processes occurring in experimental specimens, discrete models are often used to validate other model types, e.g. continuum based approaches (Grassl et al., 2014). In fracture simulations, the model can be static (Man and Van Mier, 2011; Sands, 2016) or dynamic (Frantík et al., 2013; Zhao et al., 2014; Sinaie et al., 2016); in this contribution we focus only on static models with lattices of random geometry based on Voronoi tessellation, such those used by (Gedik et al., 2011; Eliáš and Le, 2012; Kang et al., 2014).

The fine discretization of the discrete model leads to extreme computational demands, but it is necessary when it is related to the meso-scale structure of the simulated material. Too long computational time of the discrete meso-scale models prohibits their wider usage in both research and practice. Researchers devote a lot of effort to speed up the simulation, see for example the *coarse graining* method (Alnaggar and Cusatis, 2012).

This paper presents a technique for the adaptive refinement of model discretization. Without this tool, it is necessary to use fine discretization from the beginning and therefore to create computationally demanding model. If adaptive refinement is available, it allows the simulation to start with coarse discretization and refine it adaptively during the simulation run.

Adaptivity is already a well-established concept in continuum modeling. It was first used in elastic problems (Babuška and Rheinboldt, 1978; Zienkiewicz and Zhu, 1987) and later was also applied to inelastic problems with localization (Selman et al., 1997; Rodríguez-Ferran and Huerta, 2000; Patzák and Jirásek, 2004; Pannachet et al., 2010). The classical approach involves error estimation, remeshing criterion, mesh re-generation and transfer of variables onto the new mesh.

Successful attempts to introduce adaptivity into discrete models already exist (Bolander et al., 1996; Sorg and Bischoff, 2014). They are based on the adaptive replacement of a continuous model with a discrete one, though the discrete model has to have regular geometry which produces directional bias. The current contribution features adaptive refinement which is performed within the discrete model only and allows the use of irregular geometry.

The proposed algorithm works as follows. Initially, the whole domain is artificially coarsely discretized. Whenever any region of the coarse model exceeds a criterion based on the equivalent stress, the coarse discretization in its vicinity is replaced by the finer one that corresponds to the real material heterogeneity. All the nonlinear phenomena occur in the fine discretization, therefore no history variables need to be transfered onto the new model structure.

2. The random geometry of the model

The model geometry is random to avoid directional the bias that occurs in any regular structure. The domain of the modeled

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http://dx.doi.org/10.1016/j.ijsolstr.2016.09.008
0020-7683/

Nomenclature

α	Tangential/normal stiffness ratio
$\boldsymbol{\Delta}$	Displacement jump between two rigid bodies
ε	Strain
γ	Parameter controlling adaptive refinements
$\lambda, \boldsymbol{\psi}$	Eigenvalues and eigenvectors of covariance matrix
σ	Stress
$\boldsymbol{\theta}$	Rotational degrees of freedom
ξ, $\boldsymbol{\xi}$	Scaling parameter; vector of ind. standard Gaussian variables
$\boldsymbol{A}$	Matrix of rigid body motion
A	Area of contact facet
$\boldsymbol{c}$	Centroid of contact facet
$\boldsymbol{C}$	Covariance matrix
D	Damage parameter
E	Elastic modulus of contact
$\tilde{E}$	Macroscopic elastic modulus
$\boldsymbol{F}$	Forces acting on nodes
f_t	Tensile strength of contact
G_t	Fracture energy of contact in tension
H, $\widehat{H}$	Random field, Gaussian random field
L	Length of contact
l_ρ	Correlation length
$l_{\min}$	Minimum distance of random nuclei
l_f, l_c	Minimum distance in fine and coarse discretization
$\boldsymbol{n}, \boldsymbol{m}, \boldsymbol{l}$	Normal and two tangential directions
N, M, L	Indices for normal and tangential directions
$\boldsymbol{M}$	Moments acting on nodes
r_f, r_c	Radius for fine and coarse discretization
$\boldsymbol{u}$	Translational degrees of freedom
V	Volume of one rigid body
W	Dissipated energy
$\boldsymbol{x}$	Vector of coordinates

element is filled with nuclei with randomly generated positions. These nuclei are added sequentially and accepted only when the distances to previously placed nuclei are greater than the chosen parameter $l_{\min}$ (see Fig. 1). Parameter $l_{\min}$ controls the size of the discrete bodies and therefore should correspond to the size of the heterogeneities in the material (e.g. aggregate diameter). Each of the nuclei will serve as one model node bearing three translational, $\boldsymbol{u}$, and three rotational, $\boldsymbol{\theta}$, degrees of freedom (DOF).

The domain is considered to be saturated when a new nucleus is rejected for a large number of subsequent trials. Delaunay triangulation is performed providing connectivity between the nuclei. A dual diagram called Voronoi tessellation then creates the geometry of the rigid bodies (see Fig. 1). Rigid bodies have common contact facets that are perpendicular to their connections; the facet centroids are denoted $\boldsymbol{c}$ (see Fig. 2).

3. Elastic behavior

The macroscopic elastic behavior of a discrete system is *independent* of the size of the discrete units. This statement is a fundamental assumption of the adaptive technique, but is not obvious. The proof and numerical verification of this statement are delivered bellow.

3.1. Scaling of the elastic problem

This subsection is based on work of Cusatis et al. (2011b), from which it adopts both notation and theory. The equations are based on an assumption of ideally rigid bodies connected by linear-elastic

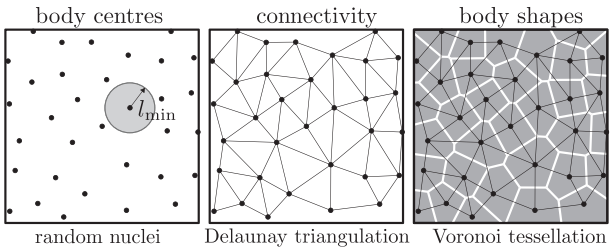

Fig. 1. Model geometry obtained by the Voronoi tessellation on randomly placed nuclei with a restricted minimum distance $l_{\min}$; 2D sketch.

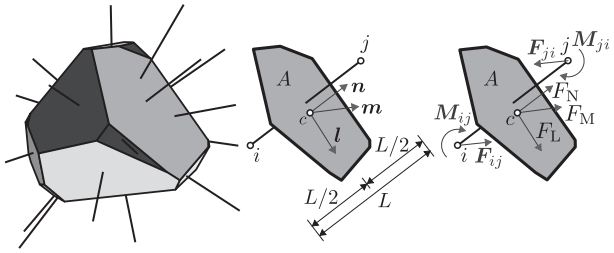

Fig. 2. One discrete body of random geometry and one contact facet between nuclei i and j – normal and tangential directions and forces.

bonds. This type of model is known in literature as the rigid-body-spring network (Kawai, 1978; Bolander et al., 2000).

Let us analyze one contact between nuclei i and j of coordinates $\boldsymbol{x}_i = [x_{1i}, x_{2i}, x_{3i}]^T$ and $\boldsymbol{x}_j$ with central point $\boldsymbol{c}$, and lenght L, see Fig. 2. The translations of point i are denoted $\boldsymbol{u}_i = [u_{1i}, u_{2i}, u_{3i}]^T$ and the rotations are $\boldsymbol{\theta}_i = [\theta_{1i}, \theta_{2i}, \theta_{3i}]^T$. Then, from the rigid body motion (assuming small rotations), the position of any point $\boldsymbol{x}$ inside the body associated with nucleus i can be expressed as

$$\boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{u}_i + \boldsymbol{\theta}_i \times (\boldsymbol{x} - \boldsymbol{x}_i) = \boldsymbol{A}_i(\boldsymbol{x}) \begin{bmatrix} \boldsymbol{u}_i \\ \boldsymbol{\theta}_i \end{bmatrix} \tag{1}$$

with matrix $\boldsymbol{A}_i(\boldsymbol{x})$ being

$$\boldsymbol{A}_i(\boldsymbol{x}) = \begin{bmatrix} 1 & 0 & 0 & 0 & x_3 - x_{3i} & x_{2i} - x_2 \\ 0 & 1 & 0 & x_{3i} - x_3 & 0 & x_1 - x_{1i} \\ 0 & 0 & 1 & x_2 - x_{2i} & x_{1i} - x_1 & 0 \end{bmatrix} \tag{2}$$

The displacement discontinuity $\boldsymbol{\Delta}_{ij}$ between bodies i and j is measured by their separation at the common facet centroid $\boldsymbol{c}$.

$$\boldsymbol{\Delta}_{ij} = \begin{bmatrix} \Delta_{1ij} \\ \Delta_{2ij} \\ \Delta_{3ij} \end{bmatrix} = \boldsymbol{A}_j(\boldsymbol{c}) \begin{bmatrix} \boldsymbol{u}_j \\ \boldsymbol{\theta}_j \end{bmatrix} - \boldsymbol{A}_i(\boldsymbol{c}) \begin{bmatrix} \boldsymbol{u}_i \\ \boldsymbol{\theta}_i \end{bmatrix} \tag{3}$$

Contact forces are calculated based on the displacement discontinuity. Three contact forces acting at point $\boldsymbol{c}$ in the normal direction, $\boldsymbol{n}$, and two tangential directions, $\boldsymbol{m}$ and $\boldsymbol{l}$, are given by

$$\begin{bmatrix} F_N \\ F_M \\ F_L \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} \boldsymbol{n}^T \boldsymbol{\Delta}_{ij} \\ \alpha \boldsymbol{m}^T \boldsymbol{\Delta}_{ij} \\ \alpha \boldsymbol{l}^T \boldsymbol{\Delta}_{ij} \end{bmatrix} \tag{4}$$

where L is the distance between nodes i and j, A is the area of the contact facet and α and E are two elastic parameters of the contact providing normal stiffness EA/L and tangential stiffness $\alpha EA/L$.

The forces and moments acting on node i ($\boldsymbol{F}_{ij}$ and $\boldsymbol{M}_{ij}$) and j ($\boldsymbol{F}_{ji}$ and $\boldsymbol{M}_{ji}$) due to contact ij can be obtained from the principle of

Please cite this article as: J. Eliáš, Adaptive technique for discrete models of fracture, International Journal of Solids and Structures (2016), http://dx.doi.org/10.1016/j.ijsolstr.2016.09.008