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An analytical solution for the transient response of a semi-infinite elastic medium with a buried arbitrary cylindrical line source

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ABSTRACT

This study develops an analytical solution for the transient response of a semi-infinite medium subjected to an arbitrary cylindrical line source buried at a certain depth by using the Laplace transform and Cagniard's method. The analytical solution is presented in a simple closed form and each term represents a transient physical wave. During the solution procedure, the arbitrary source pulse is first decomposed into a compressional wave (P) and a tangential wave (S), and then analytical solutions in the Laplace domain for the P and S pulse sources are derived. By applying Cagniard's inverse Laplace transform method and the convolution theorem, analytical solutions in the time domain are obtained. Finally, the analytical solution for arbitrary source pulse can be expressed as the superposition of the solutions for the P and S pulse sources. Numerical examples are provided to illustrate some interesting features of the cylindrical P and S wave propagation in a semi-infinite media with a free surface. A head wave can be observed in a certain area of the semi-infinite media due to the S pulse, which is not found in the P pulse problem.

1. Introduction

Wave motion in semi-infinite media, either homogeneous or inhomogeneous, has been of long-standing interest in acoustics, geophysics, seismology, electromagnetic theory and solid mechanics. Problems in solid dynamics, ultrasonic, delay lines and impact have also led to the analysis of waves in a semi-infinite medium (Graff, 1975; Pao, 1983). Lamb (1904) first studied the propagation of tremors over the surface of a semi-infinite elastic medium. He studied two exterior problems of the application of pulses on the surface and two interior problems caused by cylindrical pulses buried at a certain depth beneath the surface. Since this study by Lamb, a great many contributions have appeared in the literature, and this type of issue is commonly called Lamb's problem (Kuznetsov and Terentjeva, 2015a, 2015b).

When a cylindrical pulse is emitted from a buried line source, the subsequent disturbance at any point near the surface is much more complex than that for an incident plane pulse. This type of problem is an important issue in seismology and has been studied by many scholars. Both numerical and analytical methods are employed to solve this type of problem. The technical literature in this area is quite extensive, and the methods of analysis are quite sophisticated.

Nakano (1925) studied a buried dilatation line source in a semi-infinite medium by using the approximate evaluation of the integrals by the methods of steepest descent and stationary phase, but he did not generalize the harmonic steady-state response solutions to transient response results. Lapwood (1949) restudied this problem and gave an exact formal solution in terms of double integrals. These are evaluated approximately for the case when the depth of the source and point of reception are small compared with their distance from each other, which allows a description of the sequence of pulses that arrive at the point of reception. Murrell and Ungar (1982) illustrated the use of the differential transform technique for solving dilatation source problems. Some other numerical methods have also been used to solve this type of problem, such as that of Kuznetsov and Terentjeva (2015a, 2015b).

Until now, several analytical solutions for the buried dilatation line pulse problem have been provided in the literature. Garvin (1956) obtained an analytical solution by employing a suitable distortion of the contour given by Cagniard (1939), although this analytical solution is limited to surface displacement. Later, Mitra (1958) employed the same method to determine the surface displacements, although the character of the dilatational source is different from that in the work of Garvin (1956).

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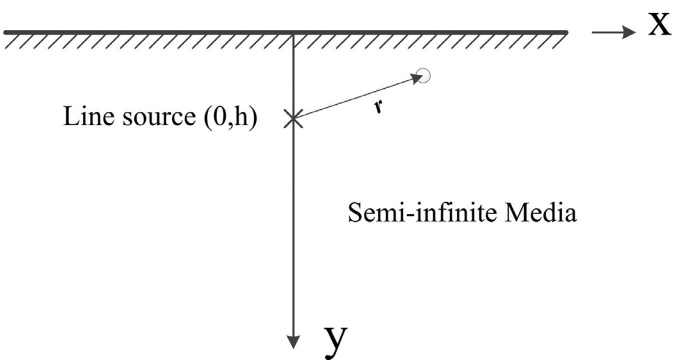

Fig. 1. Sketch map of a semi-infinite media with a buried line source.

Sànchez-Sesma and Iturraràn-Viveros (2006) revisited the classic Garvin work and extended it for virtually any given time signal. Sànchez-Sesma et al. (2013) revisited Alterman and Loewenthal (1969), which extended Garvin's solution to points in the interior of the half-space. Starting with the formulas given by Sànchez-Sesma et al. (2013) and Kausel and Martin (2014) provided the long-time results.

The above-mentioned articles only provide the analytical solutions for a cylindrical P pulse, while solutions for cylindrical S or arbitrary line sources have not been given. Only Lapwood (1949) provided an approximate solution for the S pulse problem by numerical methods. In fact, clear insight into the behaviour of the cylindrical line source problem is necessary for understanding wave propagation during an earthquake. Moreover, analytical solutions can be used to validate the correctness of various numerical techniques. Therefore, the research on wave motion in a semi-infinite elastic medium subjected to buried cylindrical line sources is of great theoretical and practical importance.

Cagniard's method can be used to invert the Laplace transform and has been widely used to solve Lamb's problems, such as in above-mentioned articles by Garvin (1956), Mitra (1958) and Alterman and Loewenthal (1969). de Hoop (1957, 1958) modified Cagniard's method for solving pulse problems, which could simplify the method considerably. Some other types of problems can also be solved by using this method, such as in de Hoop (1960, 1970), Tsai and Ma (1991), Ma and Huang (1996a, 1996b) and Ricciardello et al. (2011). This method will also be employed in this investigation.

In this paper, we provide a solution for the transient response of a semi-infinite medium due to the application of a buried cylindrical wave source, which is presented in a simple closed form where each term represents a transient physical wave. The arbitrary line source can be decomposed into a P pulse and an S pulse, which can each be calculated. The paper is organized as follows. The next section outlines the basic equations and associated boundary conditions. The excitation sources and wave fields for the P-pulse and S-pulse line sources are presented in detail in Section 3. Then, the derivations of the analytical solutions in the Laplace domain and the corresponding inverse Laplace transform are presented in Sections 4 and 5. In the sixth section, several examples are provided to illustrate some interesting features of the cylindrical P and S wave propagation in semi-infinite media. Finally, a summary of the work and some conclusive remarks close the paper.

2. Basic equations and boundary conditions

The transient response of a semi-infinite medium with a buried cylindrical line source is studied in this paper. As shown in Fig. 1, the semi-infinite medium is anchored in a Cartesian coordinate system, and the surface is along the x direction. The cylindrical line source is located at (0, h) and is along the z direction. As the line source is infinite long, this problem is analysed with plane strain model.

Without considering the body force, the displacement equation of motion in an elastic medium can be written as:

$$(\lambda+\mu)\nabla\nabla\cdot\mathbf{u}+\mu\nabla^2\mathbf{u}=\rho\frac{\partial^2\mathbf{u}}{\partial t^2} \tag{1}$$

where, $\mathbf{u}$ represents the displacement vector, ∇ is the Hamiltonian operator. λ and μ are the Lamé parameters, ρ is the density of the elastic media, and t is time.

According to the Helmholtz theorem, a vector field can be expressed as the sum of the gradient of a scalar field and the curl of a vector field.

$$\mathbf{u}=-\nabla\phi+\nabla\times\psi,\ \nabla\cdot\psi=0 \tag{2}$$

where ϕ and ψ are the scalar and vector displacement potentials, respectively. Substituting Eq. (2) into Eq. (1), we obtain

$$\begin{aligned} V_P^2\nabla^2\phi&=\frac{\partial^2\phi}{\partial^2 t},\quad V_P=\sqrt{(\lambda+2\mu)/\rho}\\ V_S^2\nabla^2\psi&=\frac{\partial^2\psi}{\partial^2 t},\quad V_S=\sqrt{\mu/\rho}\end{aligned} \tag{3}$$

where V_P and V_S are the velocities of the P and S waves.

For plane strain problem, the displacements and stresses in the semi-infinite media can be written as follows

$$\begin{aligned} u_x&=-\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y}\\ u_y&=-\frac{\partial\phi}{\partial y}-\frac{\partial\psi}{\partial x}\end{aligned} \tag{4a}$$

$$\begin{aligned}\sigma_{xx}&=(\lambda+2\mu)\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right)-2\mu\frac{\partial u_y}{\partial y}\\ \sigma_{yy}&=(\lambda+2\mu)\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right)-2\mu\frac{\partial u_x}{\partial x}\\ \sigma_{xy}&=\mu\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)\end{aligned} \tag{4b}$$

where, σ_{ij} represents the stress and u_j represents the displacement in j direction (i, j=x, y). As the surface of semi-infinite media is a free surface, both the axial and tangential stresses are zero:

$$\sigma_{yy}(x,y=0,t)=0,\ \ \sigma_{xy}(x,y=0,t)=0 \tag{5}$$

3. Analytical solutions in the Laplace domain

The Laplace transform is written as

$$\bar{m}(x,y,p)=\int_0^\infty m(x,y,t)e^{-pt}dt \tag{6}$$

where $\bar{m}(x, y, p)$ is the Laplace transform of $m(x, y, t)$. Applying the Laplace transform to Eq. (3) and boundary condition (6) with respect to time t, we have

$$\text{Wave equations:}\quad \nabla^2\bar{\phi}=\frac{p^2}{V_P^2}\bar{\phi},\ \nabla^2\bar{\psi}=\frac{p^2}{V_S^2}\bar{\psi} \tag{7}$$

$$\text{Boundaryconditions:}\quad \begin{aligned}&\rho\left(V_P^2-2V_S^2\right)\frac{\partial\bar{u}_x}{\partial x}+\rho V_P^2\frac{\partial\bar{u}_y}{\partial y}=0\\ &\mu\left(\frac{\partial\bar{u}_x}{\partial y}+\frac{\partial\bar{u}_y}{\partial x}\right)=0\end{aligned} \tag{8}$$

As there are only two types of waves in an infinite elastic medium, any type of cylindrical line source can be decomposed into a P pulse and an S pulse. Therefore, the analytical solution for

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