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Stresses produced in an orthotropic half-plane under a moving line load

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ABSTRACT

The stresses and displacements of an orthotropic half-plane were obtained under a line load moving at a constant speed over its surface. The Fourier transform was applied to solve the problem at all speeds. It was verified that the stresses and displacements can be readily classified into three regimes according to the speed of the moving load. The expressions for the stresses and displacements have shapes similar to those of the solutions to the corresponding problem for an isotropic half-plane.

1. Introduction

When a meteorite passes through the atmosphere, it acts as a moving load that stresses the surface of Earth. The velocity of the moving load is great, and infinite stresses arise on the planes that move with the moving load. Sneddon (1952) estimated the stresses produced by a line load moving at a subsonic speed on an elastic half-space using the Fourier transform. Cole and Huth (1958) estimated the stresses and displacements under a high-speed moving line load applied to Earth's surface using the complex function method for subsonic, transonic, and supersonic speeds. In their study, it was determined that at transonic and supersonic speeds, stresses are infinite on planes that move with the moving load applied to Earth's surface. However, their paper included some typographical errors, which were corrected by Georgiadis and Barber (1993) and by Rahman (2001). Atsumi and Itou (1970) estimated the stresses induced in an elastic Cosserat half-plane by a moving load for all speeds using the Fourier transform.

If the surface of Earth is not isotropic but anisotropic, the stresses can be estimated using the anisotropic theory of elasticity. Liou and Sung (2008) estimated the stresses in an anisotropic half-plane under a moving load applied over the surface of the half-plane at a subsonic speed. Later, Liou and Sung (2012) investigated the response of an anisotropic half-plane under a moving load at all speeds. Closed forms for the stresses and displacements were obtained in these two studies. However, the derivations of the equations were denoted using matrices. If explicit expressions for the stresses are required, additional troublesome calculations must be performed.

In the present study, the stresses and displacements of an orthotropic half-plane with a line load moving at a constant speed on the surface of the half-plane were estimated using the Fourier transform. For an isotropic half-plane, Cole and Huth (1958) showed that the stresses are infinite on planes with discontinuous displacements. The same behaviors have also been observed for an orthotropic half-plane at transonic and supersonic speeds.

2. Fundamental equations

The rectangular coordinates $(\bar{x}, \bar{y})$ are fixed in the half-plane $\bar{y} > 0$, as shown in Fig. 1. We considered a concentrated vertical line pressure p, defined as positive when acting downward, moving in the positive $\bar{x}$-direction at a speed U. When the problem is solved under the plane stress condition, the equations of motion can be reduced to the following form:

$$c_{11}\frac{\partial^2 u}{\partial \bar{x}^2} + \frac{\partial^2 u}{\partial \bar{y}^2} + (1 + c_{12})\frac{\partial^2 v}{\partial \bar{x}\partial \bar{y}} = \frac{1}{c_T^2}\frac{\partial^2 u}{\partial \bar{t}^2},$$
$$c_{22}\frac{\partial^2 v}{\partial \bar{y}^2} + \frac{\partial^2 v}{\partial \bar{x}^2} + (1 + c_{12})\frac{\partial^2 u}{\partial \bar{x}\partial \bar{y}} = \frac{1}{c_T^2}\frac{\partial^2 v}{\partial \bar{t}^2}, \tag{1}$$

with

$$c_{11} = \frac{E_x}{\mu_{xy}[1 - (E_y/E_x)\nu_{xy}^2]}, \quad c_{22} = c_{11}\frac{E_y}{E_x},$$
$$c_{12} = \nu_{xy}c_{22}, \quad c_T = \left(\frac{\mu_{xy}}{\rho}\right)^{1/2} \tag{2}$$

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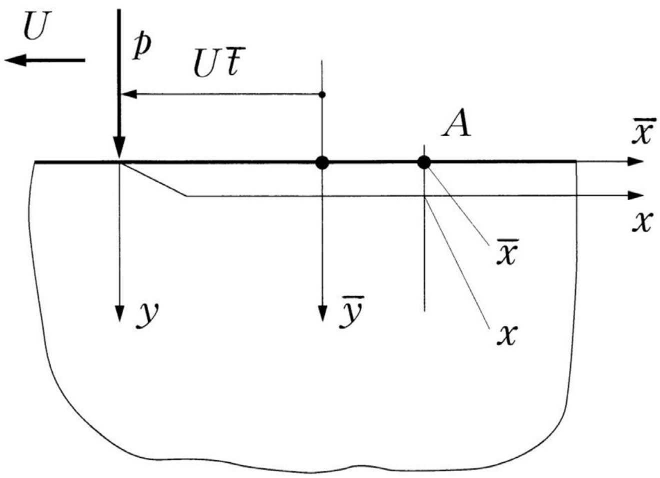

Fig. 1. Coordinate systems and moving load.

where u and v are defined as the $\bar{x}$- and $\bar{y}$-components of the displacement, respectively; $E_x$and E_y are the Young's moduli in the $\bar{x}$- and $\bar{y}$-directions, respectively; μ_{xy} is the modulus of rigidity; ν_{xy} is Poisson's ratio; ρ is the density of the material; and $\bar{t}$ is time.

To solve the problem, new coordinates $(x,\ y)$ fixed to the load were introduced, as shown in Fig. 1. Using the Galilean transformation

$$x = U\,\bar{t} + \bar{x}, \quad y = \bar{y}, \quad t = \bar{t}, \tag{3}$$

Eq. (1) can be reduced to the form

$$c_{11}\,\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + (1 + c_{12})\,\frac{\partial^2 v}{\partial x\,\partial y} - \frac{U^2}{c_T^2}\frac{\partial^2 u}{\partial x^2} = 0,$$

$$c_{22}\,\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} + (1 + c_{12})\,\frac{\partial^2 u}{\partial x\,\partial y} - \frac{U^2}{c_T^2}\frac{\partial^2 v}{\partial x^2} = 0. \tag{4}$$

The stresses can be expressed using the load-fixed coordinates $(x,\ y)$ as

$$\frac{\sigma_x}{\mu_{xy}} = c_{11}\,\frac{\partial u}{\partial x} + c_{12}\,\frac{\partial v}{\partial y}, \quad \frac{\sigma_y}{\mu_{xy}} = c_{12}\,\frac{\partial u}{\partial x} + c_{22}\,\frac{\partial v}{\partial y},$$

$$\frac{\tau_{xy}}{\mu_{xy}} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}. \tag{5}$$

The boundary conditions at the surface $y=0$ are given by

$$\sigma_y\big|_{y=0} = -p\,\delta(x), \quad \tau_{xy}\big|_{y=0} = 0, \tag{6}$$

where $\delta(x)$ is the Dirac delta function.

3. Analysis

To solve for the stresses and displacements, the following Fourier transforms were introduced:

$$\bar{f}(\xi) = \int_{-\infty}^{\infty} f(x)\,\exp(i\,\xi\,x)\,\mathrm{d}x,$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{f}(\xi)\,\exp(-i\,\xi\,x)\,\mathrm{d}\xi. \tag{7}$$

Applying Eq. (7) to Eq. (4) yields

$$\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}\,y^2} + (1 + c_{12})(-i\xi)\frac{\mathrm{d}\,\bar{v}}{\mathrm{d}\,y} - \xi^2(c_{11} - M_T^2)\,\bar{u} = 0,$$

$$c_{22}\frac{\mathrm{d}^2\bar{v}}{\mathrm{d}\,y^2} + (1 + c_{12})(-i\xi)\frac{\mathrm{d}\,\bar{u}}{\mathrm{d}\,y} - \xi^2(1 - M_T^2)\,\bar{v} = 0, \tag{8}$$

where

$$M_T = \frac{U}{c_T}. \tag{9}$$

Eq. (8) can be readily reduced to

$$\frac{\mathrm{d}^4\bar{u}}{\mathrm{d}\,y^4} + q\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}\,y^2} + r\,\bar{u} = 0, \qquad \frac{\mathrm{d}^4\bar{v}}{\mathrm{d}\,y^4} + q\frac{\mathrm{d}^2\bar{v}}{\mathrm{d}\,y^2} + r\,\bar{v} = 0, \tag{10}$$

where

$$q = \left[\frac{(c_{12}^2 - c_{11}\,c_{22} + 2\,c_{12}) + (1 + c_{22})M_T^2}{c_{22}}\right]\xi^2,$$

$$r = \left[\frac{(c_{11} - M_T^2)(1 - M_T^2)}{c_{22}}\right]\xi^4. \tag{11}$$

The characteristic equation for the differential equation given in Eq. (10) is given by

$$\lambda^4 + q\lambda^2 + r = 0. \tag{12}$$

The solutions to Eq. (12) can be expressed as

$$\lambda_n = \lambda_{n\,R}\,|\xi| + i\,\lambda_{n\,I}\,\xi \quad (n = 1,\ 2,\ 3,\ 4) \tag{13}$$

where $\lambda_{n\,R}$ and $\lambda_{n\,I}$ are real numbers. The first real number $\lambda_{n\,R}$ must be negative because stresses and displacements must approach zero as $y \to \infty$ at low moving speeds U. At high moving speeds U, displacements are discontinuous on planes in the half-plane. The planes cannot appear ahead of the line pressure, and thus $\lambda_{n\,I}$ must be positive when $\lambda_{n\,R}=0$. Therefore, λ_1 can be selected as follows:

for $0 < q^2 - 4r$ and $0 < -q + (q^2 - 4r)^{1/2}$,

$$\lambda_1 = -[-q + (q^2 - 4r)^{1/2}]^{1/2}/\sqrt{2}, \text{ and} \tag{14}$$

for $0 < q^2 - 4r$ and $-q + (q^2 - 4r)^{1/2} < 0$,

$$\lambda_1 = \{[q - (q^2 - 4r)^{1/2}]^{1/2}\,i/\sqrt{2}\}\,\mathrm{sgn}(\xi). \tag{15}$$

In Eq. (15), $\mathrm{sgn}(\xi)$ is the signum function. The second solution λ_2 to Eq. (12) can be obtained as follows:

for $0 < q^2 - 4r$ and $0 < -q - (q^2 - 4r)^{1/2}$,

$$\lambda_2 = -[-q - (q^2 - 4r)^{1/2}]^{1/2}/\sqrt{2}, \text{ and} \tag{16}$$

for $0 < q^2 - 4r$ and $-q - (q^2 - 4r)^{1/2} < 0$,

$$\lambda_2 = \{[q + (q^2 - 4r)^{1/2}]^{1/2}\,i/\sqrt{2}\}\,\mathrm{sgn}(\xi). \tag{17}$$

If the value of $q^2 - 4r$ is negative, λ_1 and λ_2 must be selected according to the value of $\cos(\theta_1/2)$ as follows:
for $0 < \cos(\theta_1/2)$,

$$\lambda_1 = -\sqrt{\eta_1/2}\cos(\theta_1/2) + i\,\sqrt{\eta_1/2}\sin(\theta_1/2)\,\mathrm{sgn}(\xi), \tag{18}$$

$$\lambda_2 = -\sqrt{\eta_1/2}\cos(\theta_1/2) - i\,\sqrt{\eta_1/2}\sin(\theta_1/2)\,\mathrm{sgn}(\xi), \tag{19}$$

and for $\cos(\theta_1/2) < 0$,

$$\lambda_1 = \sqrt{\eta_1/2}\cos(\theta_1/2) + i\,\sqrt{\eta_1/2}\sin(\theta_1/2)\,\mathrm{sgn}(\xi), \tag{20}$$

$$\lambda_2 = \sqrt{\eta_1/2}\cos(\theta_1/2) - i\,\sqrt{\eta_1/2}\sin(\theta_1/2)\,\mathrm{sgn}(\xi), \tag{21}$$

where

$$\eta_1 = 2\sqrt{r}, \quad \theta_1 = \tan_2^{-1}[(4r - q^2)/(-q)]. \tag{22}$$

In all cases, λ_3 and λ_4 are given by

$$\lambda_3 = -\lambda_1, \quad \lambda_4 = -\lambda_2. \tag{23}$$

For the lower half-plane, the solutions to Eq. (10) have the following forms in terms of the unknown coefficients A, B, C, and D:

$$\bar{u} = A\,\exp(\alpha\,y) + B\,\exp(\beta\,y), \qquad \bar{v} = C\,\exp(\alpha\,y) + D\,\exp(\beta\,y), \tag{24}$$

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