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A domain-independent interaction integral method for evaluating the dynamic stress intensity factors of an interface crack in nonhomogeneous materials

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ABSTRACT

We propose a new domain-independent interaction integral (DII-integral), which can evaluate the dynamic stress intensity factors (DSIFs) of an interface crack in nonhomogeneous materials under dynamic loading conditions. The DII-integral is rigorously proved to be domain-independent of arbitrary interfaces emerged in the integral domain. Since no material property derivative is involved in the DII-integral formulation, it can be applied to the interface crack problems with both differentiable and non-differentiable material properties. By using the extended finite element method (XFEM) combined with the DII-integral, several benchmark problems are investigated to examine the validity of the proposed DII-integral and the influence of material nonhomogeneity on the DSIFs. The results show that the present DII-integral is effective and efficient to evaluate the DSIFs of an interface crack in nonhomogeneous materials.

1. Introduction

Since advanced electronic and mechanical devices are comprised of different components, these include many complicated material interfaces. Some interfaces are nonhomogeneous with arbitrary material gradients owing to specific purposes, e.g. reduction of mismatch stress at the interface and strengthen of adhesion between adjacent layers. As the interface cracking is a key potential fracture phenomenon (Shen et al., 2001), it is essential for keeping the reliability of devices to evaluate precisely the stress intensity near the crack tip. Sometimes, the devices may meet dynamic loading, which is caused by, for example, falling down of electronic devices. Compared with static cases, the stress singularity in the vicinity of an interface crack tip is more complex due to the stress waves under dynamic loading. Hence, it is a tough issue to predict the interfacial fracture behavior in the engineering devices.

The pioneering work on interface crack problem came from Williams (1959), who founded the near-tip fields for an interface crack between isotropic bimaterials. Clements (1971) solved the interface crack problem between dissimilar anisotropic materials. Rice (1988) derived the form of near-tip fields based on the function theory, and discussed the complex stress intensity factor (SIF). Then, Suo (1990) and Wang et al. (1992) made further contributions to interface crack problems. However, the stress singularity in the vicinity of the crack tip becomes more complicated when it comes to dynamic interface cracks. Willis (1971) proposed an explicit fracture criterion by defining a ‘stress concentration vector’ to illustrate an interface crack moving uniformly between dissimilar anisotropic elastic half-spaces. Yang et al. (1991) examined the singular fields around interface crack running non-uniformly along the interface between two anisotropic materials, considering rapidly applied loads, fast crack propagation and strain rate dependent material response. However, analytical methods will be greatly limited for general interface crack problems with complicated geometries and boundary conditions.

In recent decades, many different numerical methods have been set up to deal with complicated engineering problems. Banks-Sills and Sherer (2002) developed a conservative integral for determining SIFs of a bimaterial notch. Boundary element method was explored to analyze the dynamic behavior of interface crack in two-dimensional finite bimaterial plates (Dineva et al., 2002; Beyer et al., 2008; Lei et al., 2008; Wünsche et al., 2009). A scaled boundary finite-element method was also developed to evaluate the dynamic stress intensity factors (DSIFs) of the rectangular bimaterial plate with a central interface crack (Song and Vrcelj, 2008; Song et al., 2010).

The interaction integral method is also a feasible way to evaluate the components of the mixed-mode SIFs which are critical in understanding the interface resistance to fracture (Stern et al., 1976; Kim and Paulino, 2005; Yu et al., 2009; Guo et al., 2012 and 2015). However, the related works on the interface crack problems

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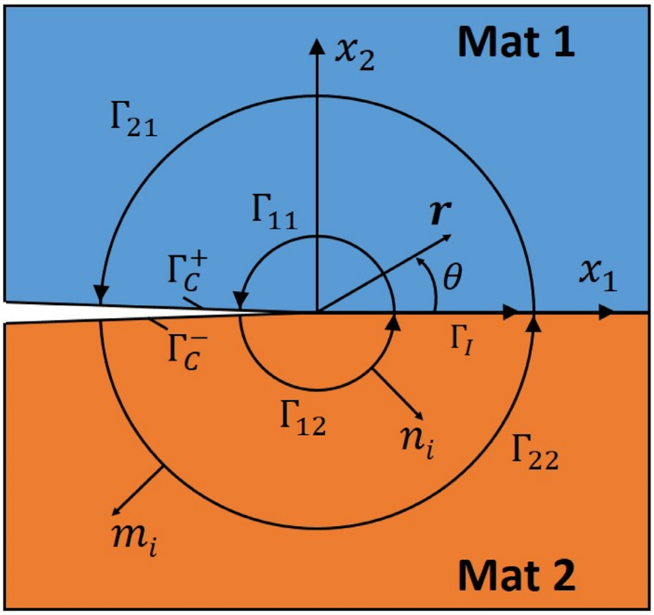

Fig. 1. Schematic of the integral domains. (The domains A_0 and A are enclosed by Γ_0 and Γ_2, respectively. The domains are divided by the bimaterial interface, i.e., $A_0 = A_{01} + A_{02}$, $A = A_1 + A_2$. Here, $\Gamma = \Gamma_{11} + \Gamma_{12}$, $\Gamma_0 = \Gamma_2 + \Gamma_C^+ + \Gamma^- + \Gamma_C^-$, $\Gamma_2 = \Gamma_{21} + \Gamma_{22}$).

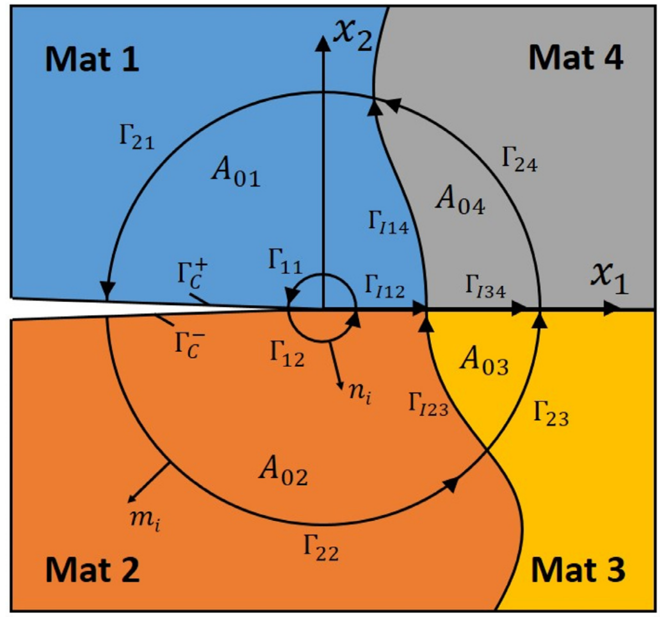

Fig. 2. Schematic of integral domains containing an arbitrary curved interface. (The domain A is divided by the interfaces Γ_I and Γ_{I1} into four subdomains: A_{01}, A_{02}, A_{03} and A_{04}, which are enclosed by contours Γ_{01}, Γ_{02}, Γ_{03} and Γ_{04}, respectively. $\Gamma_{01} = \Gamma_{21} + \Gamma_C^+ + \Gamma_{11}^- + \Gamma_{I12} + \Gamma_{I14}$, $\Gamma_{02} = \Gamma_{22} + \Gamma_{I23} + \Gamma_{I12}^- + \Gamma_{12}^- + \Gamma_C^-$, $\Gamma_{03} = \Gamma_{23} + \Gamma_{I34}^- + \Gamma_{I23}^-$ and $\Gamma_{04} = \Gamma_{24} + \Gamma_{I14}^- + \Gamma_{I34}$; $\Gamma_I = \Gamma_{I12} + \Gamma_{I34}$ and $\Gamma_{I1} = \Gamma_{I23} + \Gamma_{I14}$).

are relatively rare, especially for the dynamic case. Matos et al. (1989) and Yu et al. (2010) proposed different interaction integral methods to calculate the SIFs in homogeneous and nonhomogeneous materials, respectively. Both of them were concerned with static cases. By extending the interaction integral method, Lo et al. (1994) investigated several features of the characteristics of a dynamic interface crack. However, the materials on both sides of the interface are still limited to homogenous. To our best knowledge, the interface crack problems in nonhomogeneous materials under dynamic loading conditions are still rarely investigated, especially for the cases with complicated interfaces. Hence, we develop a new domain-independent interaction integral (DII-integral) method for evaluating the DSIFs of an interface crack in nonhomogeneous materials under dynamic loading conditions.

This paper is organized as follows. In Section 2, the DII-integral formulation without material property derivative is derived, which permits the material properties to be discontinuous and non-differentiable; and then, the domain-independence of the present DII-integral is proved rigorously. Section 3 briefly introduces the numerical implementation of the DII-integral combined with the extended finite element method (XFEM) and the Newmark's method used for the time integration. In Section 4, several benchmark examples are explored to verify the effectiveness and efficiency of the present DII-integral; and then, the influence of material nonhomogeneity on the DSIFs is investigated. Finally, some conclusions are drawn in Section 5.

2. DII-integral for an interface crack problem in nonhomogeneous materials

2.1. Auxiliary fields for an interface crack between two nonhomogeneous media

The in-plane stresses directly ahead of the crack tip ($\theta = 0$) induced by the singular near-tip fields of an interface crack problem can be given by (Yu et al., 2010)

$$\sigma_{22} + i\sigma_{12} = \boldsymbol{K}(2\pi r)^{-1/2}(r/l)^{i\epsilon^{tip}} \tag{1}$$

where $\boldsymbol{K} = K_1 + iK_2$ denotes the complex SIF; $i = \sqrt{-1}$; l is the characteristic length; and the bimaterial constant ε^{tip} takes the form

$$\epsilon^{tip} = \frac{1}{2\pi}\ln\left(\frac{\kappa_1^{tip}/G_1^{tip} + 1/G_2^{tip}}{\kappa_2^{tip}/G_2^{tip} + 1/G_1^{tip}}\right) \tag{2}$$

where the Kolosov constants are $\kappa_m^{tip} = 3 - 4\nu_m^{tip}$ for plane strain, and $\kappa_m^{tip} = (3 - \nu_m^{tip})/(1 + \nu_m^{tip})$ for plane stress; G_m^{tip} and ν_m^{tip} are shear modulus and Poisson's ratio at the crack tip, respectively; $m = 1, 2$. To determine the proportion of the shear traction to the normal traction, a phase angle ψ is introduced as (Rice, 1988)

$$\psi = \tan^{-1}\left(\frac{\mathrm{Im}\left[\boldsymbol{K}(r/l)^{i\epsilon^{tip}}\right]}{\mathrm{Re}\left[\boldsymbol{K}(r/l)^{i\epsilon^{tip}}\right]}\right) \tag{3}$$

where Im[(•)] and Re[(•)] represent the imaginary and real parts of a complex (•), respectively.

To derive the DII-integral, the appropriate auxiliary fields, apart from the actual fields, are also required. For a propagating interface crack, Yang et al. (1991) have proposed an available analytical fulfilled solution. As the propagating velocity $v \to 0$, the limiting solution converges to the corresponding quasi-static solutions. Hence, for the present interface crack problems, an incompatibility formulation is adopted as (Sukumar et al., 2004)

$$u_i^{aux} = \begin{cases} \dfrac{f_i^{\mathrm{I}}\left(r,\ \theta,\ \epsilon^{tip},\ \kappa_m^{tip}\right)}{4G_m^{tip}\cosh\left(\pi\epsilon^{tip}\right)}\sqrt{\dfrac{r}{2\pi}} & (K_1^{aux} = 1,\ \ K_2^{aux} = 0) \\ \dfrac{f_i^{\mathrm{II}}\left(r,\ \theta,\ \epsilon^{tip},\ \kappa_m^{tip}\right)}{4G_m^{tip}\cosh\left(\pi\epsilon^{tip}\right)}\sqrt{\dfrac{r}{2\pi}} & (K_1^{aux} = 0,\ \ K_2^{aux} = 1) \end{cases} \tag{4}$$

$$\sigma_{ij}^{aux} = C_{ijkl}^{tip}\varepsilon_{kl}^{0} = C_{ijkl}^{tip}\left(u_{k,l}^{aux} + u_{l,k}^{aux}\right)/2 \tag{5}$$

$$\varepsilon_{ij}^{aux} = S_{ijkl}(\mathbf{x})\sigma_{kl}^{aux} \tag{6}$$

where $(\bullet)^{aux}$ denotes corresponding auxiliary variable; ε_{kl}^0 denotes a temporary strain tensor; $S_{ijkl}(\mathbf{x})$ is the compliance tensor at point

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