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Periodic response predictions of beams on nonlinear and viscoelastic unilateral foundations using incremental harmonic balance method

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ABSTRACT

Buildings, railway tracks, drill strings and off-shore pipelines are all treated as structures on elastic foundations in order to study their response behavior in many engineering applications. Also, flexible polyurethane foams used for cushioning in furniture, foot-ware, and automotive industries serve as foundations, and exhibit complex nonlinear viscoelastic behavior. It is challenging to develop models of systems that include these foam-like materials and are able to predict the behaviour over a wide range of loading conditions. Even when using the simpler models commonly utilized in the literature, it is computationally expensive to predict the steady-state response of these structures to static and harmonic loads. In this work a pinned-pinned beam interacting with a viscoelastic foundation which can react both in tension and compression, or in compression alone is considered. The model developed here is capable of predicting the response to static as well as dynamic forces, whether they are concentrated or distributed. If the foundation reacts only in compression, the contact region changes with beam motion and the estimation of the unknown contact region is embedded into the iterative solution procedure. The steady-state solution is expressed as the sum of an arbitrary number of modes of an undamped pinned-pinned beam and Galerkin method is used to derive equations for the modal amplitudes. Incremental harmonic balance is used to make the steady-state frequency response predictions more efficient and a pseudo arc-length continuation technique is used to track both stable and unstable solution branches. By using these computationally efficient solution approaches, it is possible to explore a much wider variety of loading conditions and also quickly determine the number of modes required for convergence of the periodic solution. By using this solution method, the influence of various system parameters on the response of the beam is studied.

1. Introduction

Nonlinear and viscoelastic behaviour is exhibited by materials such as flexible polyurethane foam used for cushioning in the furniture and automotive industries, and building soils and biological materials. To design systems that incorporate these materials, it is necessary to be able to understand and predict the static and dynamic behaviour of these systems. The focus of this work is on understanding the response of a beam interacting with a viscoelastic material.

Several foundation models were investigated in the literature (Dutta and Roy, 2002). In a Winkler foundation, the supporting medium is modelled as a system of infinitesimally close springs that produce a reaction force that is proportional to the beam displacement (Hetenyi, 1946). This is the material modelling approach adopted in this study. The choice of beam model depends on the problem being studied. For example, while at low frequencies (<500 Hz) both the Euler-Bernoulli and Timoshenko beam models give similar results, at higher frequencies the latter model tends to be more accurate (Ruge and Birk, 2007). Because most of our interest is in the low frequency behavior (<100 Hz), an Euler-Bernoulli beam model is chosen.

The most commonly used beam-foundation models allow for both compressive and tensile stresses to exist across the interface between the beam and the foundation (*bilateral foundation*). In contrast, if a foundation reacts to compressive forces but not to tensile forces, such a foundation is referred to as a *tensionless* or a *unilateral* foundation. This type of contact is more common in real world problems. With railroad tracks on the soil foundation being one of the primary motivations for research in this area, static and dynamic behaviour of infinite beams on elastic and viscoelastic

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foundations has been studied by a number of investigators (Choros and Adams, 1979; Kargarnovin et al., 2005; Tsai and Westmann, 1967; Weitsman, 1970, 1971; Younesian et al., 2006). For these cases, the beam is assumed to be infinite and so any applied point load can be considered to be at the center of the beam and all the results emanate from the inherent symmetry of the problem.

Ansari et al. (2010) studied the vibration of a finite Euler-Bernoulli beam, supported by a non-linear viscoelastic foundation traversed by a moving load. They examined the behaviour of the harmonics in the responses to sinusoidal excitation over a range of frequencies by using the method of multiple time scales. Mamandi et al. (2012) studied the dynamics of a simply supported Euler-Bernoulli beam resting on a nonlinear elastic foundation using nonlinear normal modes under three-to-one internal resonance condition. Chaotic dynamics of a finite beam on Winkler-type soil (Lenci and Tarantino, 1996) and the nonlinear dynamic behaviour and instabilities of a beam under harmonic forcing (Santee and Gonzalves, 2006) are some examples of the large body of research on finite beams on *bilateral* foundations. However, in many of applications, for example, cushioning (seat-occupant systems), adhesion between the foundation and other elements in the system is not assured and so an assumption of bilateral behaviour is not appropriate.

Studies on the behaviour of finite and semi-infinite beams on tensionless foundations are relatively rare in the literature. Predicting the response of finite beams supported by a tensionless foundation is complicated by: (a) the need to determine the contact region, and (b) the lack of symmetry in the problem. For finite beams, one of two approaches are typically adopted in these studies. In the first approach, referred to here as Method A, the local boundary conditions are applied while ensuring continuity in the regions of contact and non-contact between the beam and the foundation. Equations are solved giving the exact solution when the foundation is assumed to have linear elastic properties. In the second approach, referred to here as Method B, the global boundary conditions are applied and contact functions are defined based on the solution. In this case the solution is approximated by using Galerkin/Ritz methods.

Investigators that used the Method A solution procedure (Coskun and Engin, 1999; Silveira et al., 2008; Zhang and Murphy, 2004) dealt with simple loads giving rise to one or two contact regions. With Method A, as the number of contact and non-contact regions increase, the number of governing partial differential equations increase each with their own set of local boundary conditions and continuity equations. If the loads are dynamic, the contact region will change differently in every cycle making Method A very difficult to apply. Thus, Method B (Celep et al., 1989; Coskun and Engin, 1999; Silveira et al., 2008) was chosen as the solution procedure in this research because of the flexibility in handling a wider variety of loading conditions/deflection shapes. In this technique, it is possible to automatically handle multiple changing contact regions and complicated static loads. If the foundation is nonlinear in addition to being unilateral, the solution is obtained by perturbation methods like the method of multiple time scales (e.g., see Lancioni and Lenci (2007, 2010); Younesian et al. (2013)).

It was observed that direct time integration of the governing equations is computationally costly, particularly when damping is low, the system takes a long time to reach steady-state. Therefore, in this work, steady-state response predictions were made much more computationally efficient by using incremental harmonic balance method. Incremental harmonic balance is an iterative method that improves on an initial guess of the harmonic balance solution. The improvement is accomplished by using systematic linearized equations around an approximate periodic solution with desired number of harmonics. The iterations can be continued a desired number of times to achieve desired accuracy in the overall periodic solution. When the steady-state solution over a range of frequencies is of interest, the solution at a nearby frequency is used to develop the initial guess for the solution at the new excitation frequency. This reduces the number of iterations required to obtain an accurate solution and thereby speeds up the computation. A number of researchers from different engineering fields have used this method. Examples include Raghothama and Narayanan (2000) who used this method to study the bifurcation behavior of an articulated loading platform, and Cheung et al. (1990) who used this method to analyze multi-degree of freedom systems with cubic nonlinearities. Leung and Chui (1995) improved the performance of the incremental harmonic balance method by using a fast Fourier transform approach to compute the solution for the increments in each iteration. In recent investigations, the incremental harmonic balance method was used to study the nonlinear vibration of axially moving beams by Sze et al. (2005), and also to study the nonlinear vibration of a curved beam subject to uniform base harmonic excitation with both quadratic and cubic nonlinearities by Huang et al. (2011).

To the authors' knowledge, no one has used incremental harmonic balance method to study the steady-state periodic response of structures interacting with elastic and viscoelastic foundations. However, researchers like Kong et al. (2015) have studied the steady-state response of simpler systems like a linear bearing whose stiffness is characterized by a piecewise-nonlinear function by using a multi-term incremental harmonic balance method. In their work and many others who studied the dynamic response of systems with unilateral contact, for e.g., one-way clutches, bearings, gear backlash, it is possible and convenient to model the system as $1-, 2-$, or $3-$ degree of freedom systems with piecewise linear or nonlinear stiffness and/or damping (Blankenship and Kahraman, 1995; Shen et al., 2008; Theodossiades and Natsiavas, 2000; Wang and Zhu, 2015; Wolf et al., 2004; Xu et al., 2002; Zhu and Parker, 2005). A more comprehensive and thorough mathematical treatment of dynamics of discontinuous and non-smooth dynamical systems can be found in the works of Leine et al. (2000), and Leine and Nijmeijer (2013). Not all these studies use incremental harmonic balance method, but whichever method they use, they are interested in response behavior around one or two frequencies and the contact function defining their unilateral behaviour is a function of a single dependent variable which is just a function of time. Even though the degrees of freedom in most of these problems are not more than three, they are all complex systems exhibiting interesting behaviour at the same time from the nonlinear dynamics point of view. However, when we would like to study the response of structures interacting with a unilateral foundation, there will be space dependence in addition to time dependence.

In this paper, the use of incremental harmonic balance to predict the steady-state response behaviour of a pinned-pinned beam on a nonlinear viscolelastic foundation when subjected to harmonic loads is described. The incremental harmonic method is coupled to a parameter continuation technique to develop nonlinear frequency response of the system. The methodology proposed here can be extended to study the steady-state response of other beam-like structures and is valid for both bilateral and unilateral foundation cases.

2. Beam-foundation model

A homogeneous pinned-pinned beam of length $2L$ on a Winkler-type nonlinear, viscoelastic and unilateral foundation is considered (Bhattiprolu et al., 2013), as illustrated in Fig. 1. In the illustration, concentrated loads are applied at $x_1 = -L/2$ (a static load denoted by F_{10}), $x_2 = 0$ (a combined static and dynamic load denoted by $F_{20} + F_{2t}(t)$), and $x_3 = +L/2$ (denoted as F_{30}). The