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The relevance of transverse deformation effects in modeling soft biological tissues

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ABSTRACT

Hyperelastic constitutive models for anisotropic biological materials are frequently based on orthotropic incompressible stored energy functions. The material parameters of these models are then obtained through an optimization procedure as to fit some stress-strain experimental data. For example, in arterial wall mechanics the material data usually employed for the Holzapfel-Gasser-Ogden and the Gasser-Ogden-Holzapfel models are two uniaxial tension curves from circumferential and axial specimens. The transverse strains from these specimens are frequently not taken into consideration. In this paper we analyze the evolution of those strains, showing that an unrealistic behaviour may be predicted. We then show how transverse strains may be prescribed using our What-You-Prescribe-Is-What-You-Get (WYPIWYG) model in a very intuitive way, still capturing the longitudinal stress-strain behavior in an exact manner without employing any constitutive parameter. This is possible because, in contrast to what it is usually done, we exactly solve the equilibrium and compatibility equations without imposing the shape of the stored energy function. Furthermore, we show that the small strains formulation is naturally recovered and that the physical insight from the infinitesimal theory is preserved. In fact, for incompressible materials, the present approach can be considered as a natural extension of the infinitesimal continuum elastic framework to large strains. This new physical insight clearly shows that if some subclasses of orthotropic incompressible material models are determined with just two uniaxial curves, then the transverse behavior should be contrasted with additional experimental observations.

1. Introduction

In contrast to the usual approach employed in fiber-reinforced industrial composites, where a linear small strain analysis is performed (Jones, 1975), in soft biological tissues large strain measures and associated constitutive equations are employed because the behavior is highly nonlinear. Then, the first modelling approach to soft biological (collagen-reinforced) tissues is to employ hyperelastic constitutive models (Fung, 1993b; Humphrey, 2013). However, hyperelastic (true elastic, path independent) behavior puts some restrictions in the constitutive equations which, for the linear model, are simply fulfilled by the symmetries of the constitutive tensor. In order to also automatically fulfill these restrictions in the nonlinear regime, authors typically propose a stored energy function shape; the shape being modulated by some material parameters as to fit (with variable success) some experimental data, see Ogden (1997). For isotropic materials, many constitutive models have been proposed (see Arruda and Boyce (1993); Blatz and Ko (1962); Mooney (1940); Ogden (1972); Rivlin (1948); Yeoh (1990) among others), and usually, the linear infinitesimal theory may be recovered and the infinitesimal moduli easily related to the material parameters as, for example, in the Ogden model (Ogden, 1972).

In the case of transverse isotropic and orthotropic materials, several proposals are also available and frequently used in soft biological tissues, some of them phenomenological in approach as the popular Fung model (Chuong and Fung (1983); Fung et al. (1979), see also Humphrey (1995)), and some using microstructure information, see for example Ateshian et al. (2009); Bischoff et al. (2002); Driessen et al. (2005); Flynn and Rubin (2012); Flynn et al. (2011); Gasser et al. (2006); Holzapfel et al. (2000, 2015); Humphrey et al. (1990a); 1990b); Humphrey and Yin (1987); Lanir (1983); Li and Robertson (2009); Limbert (2011); Sacks (2003); Weiss et al. (1996). In these models a frequent approach to preserve frame invariance in general is to establish the stored energy as a function of Spencer's pseudo-invariants of the right Cauchy-Green metric tensor (Spencer, 1984). Then, to arrive at workable

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formulations, some of these invariants are frequently neglected Holzapfel (2000), see discussion in Holzapfel and Ogden (2009), and because the meaning of some of these invariants is not intuitive, the consequences may be unexpected, as we show below. Moreover, it is usual the case where the material parameters are also of difficult interpretation, and the problem encountered in finding the best solution for isotropic materials (Latorre et al., 2016; Ogden et al., 2004), is magnified in anisotropic ones (Fung, 1993a; Holzapfel et al., 2000). Then, material symmetries congruency may not be obtained either theoretically or computationally (Latorre and Montáns, 2015b), so the predictions for an isotropic material characterized with these models may result in that of an anisotropic material. Some requirements regarding this issue for stored energies based on Cauchy–Green invariants may be found in the paper of Murphy (2014).

Furthermore, a relevant point recently analyzed also by Murphy (2014) is that the proposed anisotropic models do not recover the full infinitesimal orthotropic theory. In fact, for someone not acquainted with biomechanics constitutive models but with fiber-reinforced composites background, it may be surprising that an incompressible orthotropic material may be fully characterized with just two tension curves (Holzapfel et al., 2015; Sáez et al., 2016), when the linear fiber-reinforced elastic counterpart needs six independent moduli (i.e. six slopes of six independent curves), see Holzapfel and Ogden (2009). Then, in these models the effects of transverse strains are either neglected, ignored or assumed to be correctly given *by the model* (but usually never verified) and the result may be rather unexpected (Gasser et al., 2006). It is obvious, even from the infinitesimal theory, that transverse strains are very important in 3D or 2D constrained analyses, so if transverse strains are not correctly captured, in general it is to be expected that the longitudinal behavior will neither be correctly predicted in a configuration different from the uniaxial test. General requirements on stored energies based on Cauchy–Green invariants in order to recover the full infinitesimal theory are given in Murphy (2014). A consequence of not fulfilling these requirements may be the inability of the model to properly represent a general incompressible orthotropic material not only under infinitesimal strains but also under large deformations. Indeed, for the case of arteries, it has been recently shown that predictions of Poisson's ratio with some models initially developed for these type of biological materials are inconsistent with experimental data (Skacel and Bursa, 2016). In summary, apart from the challenging interpretation of the invariants and material parameters being used (which are usually obtained from optimization procedures), and assuming that the compression behavior of fibers is properly accounted for (Latorre and Montáns, 2016b), the following difficulties are frequently found in these models: non-uniqueness of material parameters (Fung, 1993a; Holzapfel et al., 2000; Ogden et al., 2004), inconsistency with the equivalent full linear theory (Murphy, 2014), lack of numerical material symmetries congruency (Latorre and Montáns, 2015b) and unrealistic transverse strains (Skacel and Bursa, 2016).

In order to overcome all these difficulties, we have recently presented two hyperelasticity models in the realm of the WYPIWYG (What-You-Prescribe-Is-What-You-Get) approach, an approach which we also applied to viscoelasticity (Latorre and Montáns, 2015a, 2016a) and damage (Miñano and Montáns, 2015). One of the hyperelastic models is for transverse isotropic materials (Latorre and Montáns, 2013), see also Ref. Romero et al. (2016), and the other one for orthotropic materials (Latorre and Montáns, 2014b). The formulations are based on the isotropic incompressible model of Sussman and Bathe (2009), which is already available in the commercial finite element code ADINA (2012). These models are purely phenomenological, i.e. no information about the structure of the material is employed, except for the material symmetries. Furthermore, the models use meaningful, easy-to-interpret invariants of the logarithmic strains Latorre and Montáns (2014a), which are a natural extension of infinitesimal strains. In fact, we show here that the formulation itself may be understood as a natural extension of the infinitesimal model to large strains because we use the same uncoupled form for the stored energy as that of the infinitesimal framework and because logarithmic strains have unique properties parallel to those of infinitesimal strains. For example, they are additive if the principal strain directions remain fixed, the push-forward and pull-back operations are performed with the rotation part of the deformation gradient (i.e. they preserve the metric with respect both the reference and the current configurations) and the deviatoric and volumetric operators are the same as those of the infinitesimal theory. Discussions on the properties of logarithmic strains and their relation with infinitesimal strains can be found in Fiala (2015); Latorre and Montáns (2014a, 2015c); Neff et al. (2015); Xiao et al. (1997), among others. As a result, six curves (including the compression part when applicable, see Latorre et al. (2016)) are needed to fully characterize our orthotropic model. These curves are exactly captured, close to the machine precision, with the modified algorithm given below, and uniquely determine the six reference elastic moduli and their evolution. With the previous algorithm, presented in Latorre and Montáns (2014b), the solution was not obtained in an strictly exact manner (computationally speaking), although the solution could be considered exact for practical purposes if the tendency of the transverse deformations, e.g. linear, quadratic, etc., is known from experimental measurements (Skacel and Bursa, 2016). However, the new algorithm yields, if desired, numerically exact solutions which will be useful for the current analysis and without employing user-prescribed starting values. Furthermore, as we also show below, insight into the behavior of the model under finite strains is naturally and accurately obtained by an analysis of the infinitesimal theory. This insight is also crucial in order to be able to prescribe the possibly unknown (in a quantitative manner) transverse finite strains such that the behavior is according to what one would physically expect (in a qualitative manner). In our opinion, this is a much better option for unknown curves than to simply ignore them.

The rest of the paper is structured as follows. We first analyze the transverse strains under uniaxial tests obtained, just as examples, by two typical models: the Holzapfel-Gasser-Ogden model (Holzapfel et al., 2000) and the Gasser-Ogden-Holzapfel model (Gasser et al., 2006) . However we emphasize that similar situations may be present in other simplified models when transverse strains are not explicitly considered, as the experiments of Skacel and Bursa (2016) show. Then we explain the new computational procedure for the Latorre-Montáns model (Latorre and Montáns, 2014b) which exactly captures the prescribed experimental nonlinear curves, a procedure which uses the insight given by the infinitesimal model. Furthermore, we then show that this insight allows for the prediction beforehand of additional transverse strain distributions and also their influence in the third direction, showing a behavior under finite strains similar to that of the small strains theory. We finally demonstrate that the same axial and circumferential stress predictions over the arterial wall may be exactly obtained with different stored energies if different transverse strains are prescribed; i.e. a real, general orthotropic incompressible material is not correctly characterized in preferred material directions using only two curves, even if they are exactly captured, as it is immediately deduced from the linear theory. Only particular cases of orthotropy may be represented with those stored energies. Then, the adequacy of those hypothesis to represent the actual material behavior of the specific materials at hand should be tested (Holzapfel and Ogden, 2009; Skacel and Bursa, 2016).