International Journal of Solids and Structures 000 (2016) 1–13

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Higher-order beam model for stress predictions in curved beams made from anisotropic materials

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ARTICLE INFO

Article history:
Received 3 February 2016
Revised 19 May 2016
Available online xxx

Keywords:
Curved multilayered beams
Higher-order modeling
Interlaminar stresses
Anisotropic materials

ABSTRACT

A higher-order beam model for analyzing the flexural response of curved multilayered beams with constant curvature and arbitrary constant thickness is developed. The new model is derived from the Hellinger–Reissner mixed variational statement and predicts inherently equilibrated 3D stresses from an equivalent single-layer model. As a starting assumption, the hoop stress is formulated as a series of higher-order stress resultants multiplied by Legendre polynomials. The governing equations are derived in a generalized manner such that the modeling order can be adjusted and is not defined a priori. Hence, the highest order Legendre polynomial determines the modeling order. The through-thickness shear and normal stresses are derived by integrating the generalized hoop stress in Cauchy's polar equilibrium equations. As a result, all stress fields are based on the same set of variables, thereby considerably reducing the computational effort. The three stress fields, and two displacements in the radial and hoop directions are used in the Hellinger–Reissner functional to derive a new set of stress-displacement relations. The enforcement of the classical membrane and bending equilibrium equations of curved beams in the Hellinger–Reissner functional guarantees that all interlaminar and surface traction equilibrium conditions are satisfied exactly. A validation study of a composite laminate using a high-fidelity 3D finite element model shows that the stresses are captured very accurately by the present model, but with much less computational effort than the finite element model. As a result, the developed model can provide rapid and accurate insights into the expected damage onset behavior of curved laminates.

1. Introduction

Due to their lightweight structural properties, composite materials are increasingly used in primary aircraft structures. In areas of high curvature, as for example in T-shaped stringers of aircraft wings, interlaminar stresses are pronounced and can lead to premature delamination failure (Wimmer et al., 2009; Wisnom and Jones, 1995). To mitigate this debonding failure, it is crucial to robustly predict the stress fields in these curved laminated structures (Most et al., 2015; Uyar et al., 2015; Wisnom, 2012).

Recently, Most et al. (2015) emphasized the importance of accurate stress prediction for delamination failure in thick curved composites. The researchers compared different models in the literature and concluded that finite element (FE) models are able to predict stress fields accurately, but with enormous computational costs, whereas models based on first-order shear theory are easy and fast to use, but are not capable of predicting stresses with high accuracy. Therefore, there is a need for new models that capture the higher-order structural effects that influence through-thickness shear and through-thickness normal stresses, but are economical in terms of computational resources.

Some models in the literature predict interlaminar stresses in curved beams based on the first-order shear deformation theory (Mcrobbie et al., 1995; Shenoi and Wang, 2001). These models typically derive a closed-form solution, which is economical in terms of computational cost, but do not take into account higher-order phenomena, and hence are only accurate for very thin laminates (Kant and Swaminathan, 2000). Kress et al. (2005) derived a model to analyze radial stresses in moderately thick curved laminates. Roos et al. (2007) improved this model by taking into account the interlaminar shear stresses. The model provides a closed-form solution, and hence does not need a lot of computational effort. However, in comparison with 3D FE, the model predicts the stresses only to a certain accuracy. 3D FE can provide accurate stress fields predictions, but they are based on partial

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http://dx.doi.org/10.1016/j.ijsolstr.2016.08.004
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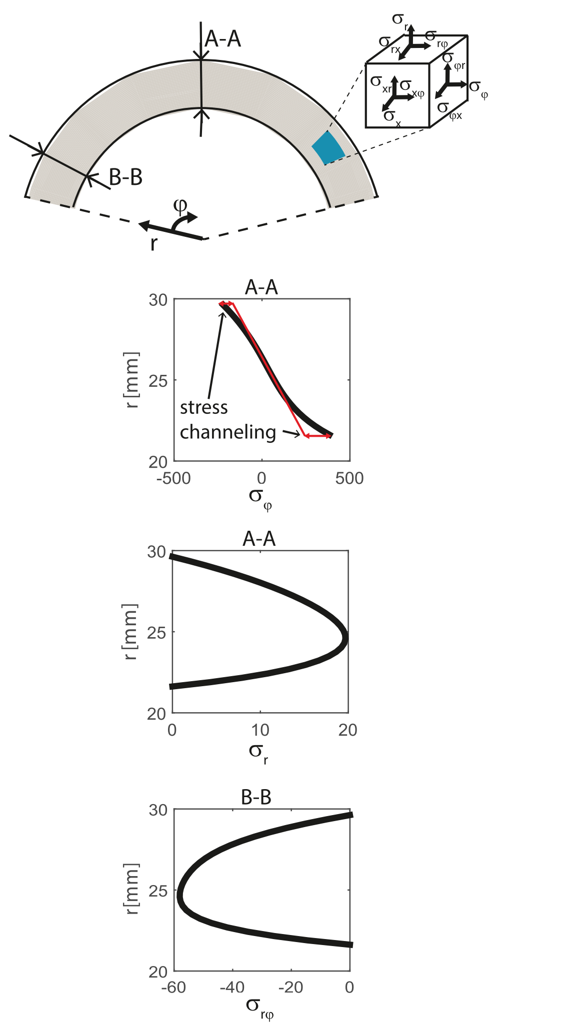

Fig. 1. Illustration of stress distribution in curved beams. The hoop stress shows stress channeling which means that the stress distribution is cubic rather than linear towards the surfaces.

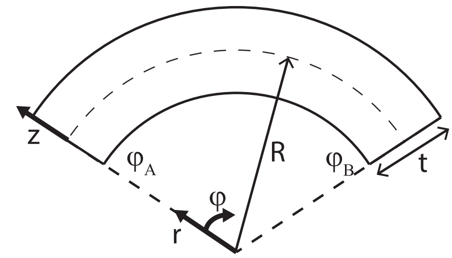

Fig. 2. Definition of a curved beam element.

differential equations which are typically costly to solve (Kant and Swaminathan, 2000). Fraternali and Bilotti (1997) suggested an FE model based on the Principle of Virtual Displacements (PVD) where each layer is modeled as a beam element and individual layers are bonded together with a constraint enforced via a penalty method. Gonzalez-Cantero et al. (2015) proposed a semi-analytical method to calculate interlaminar stresses in curved beams with constant curvature. Other authors conducted experiments to verify the real behavior of curved laminates (Hao et al., 2012; Roos et al., 2007), but these tests cannot serve as a general strategy to predict delamination failure.

An example stress distribution in a curved beam is shown in Fig. 1. We expect an accurate model to be capable of predicting the hoop stress σ_φ considering higher-order effects such as stress channeling, i.e. a cubic rather than linear stress variation towards the top and bottom surfaces (Everstine and Pipkin, 1971), as observed in Fig. 1. Further, the model needs to be able to recover the through-thickness stresses $\sigma_{r\varphi}$ and σ_r correctly, as these are crucial for the prediction of delamination failure. These transverse stresses need to fulfill the interlaminar stress continuity through the thickness and equilibria of tractions on the surfaces (Carrera, 2002). All these requirements are fulfilled in the model we present herein.

The objective of this paper is to present a new higher-order beam model for laminated curved beams. In Section 2 the derivation of the governing equations is presented. First, the hoop stresses are expressed as a generalized Taylor series expansion of higher-order stress resultants multiplied by Legendre polynomials. The transverse stress fields are then derived by integrating the hoop stress in Cauchy's equilibrium equation written in polar coordinates. Next, the Hellinger–Reissner mixed variational statement is used to derive new higher-order stress-displacement relations that, alongside the classical membrane and bending equations of a curved beam, encompass the governing field equations of the theory. The use of this formulation is inspired by a model for flat composite beams presented by Groh and Weaver (2015b). The end of Section 2 details how the governing equations are solved using the pseudo-spectral differential quadrature method. The results of the new model are compared with a high fidelity 3D FE model in Section 3. Further, the results are compared with closed-form first-order models by Timoshenko (1951) and Kedward et al. (1989). A convergence study is presented to obtain the modeling order required to calculate accurate stress fields for the laminations and geometries studied herein. Finally, the accuracy of the stress fields computed by the developed Hellinger–Reissner model and the 3D FE model are assessed by means of the residual within Cauchy's polar equilibrium equations.

2. Theory

2.1. Derivation of the governing equations

Fig. 2 shows a curved beam element with constant curvature. We use a polar coordinate system with hoop coordinate φ and radial coordinate r to derive the higher-order curved beam model. R denotes the mid-plane radius and t the thickness of the beam. The local through-thickness coordinate z is calculated as $z = r - R$ and the differential increment is $dz = dr$. In this section the derivation of the governing equations of the higher-order beam model for curved beams is described. The procedure outlined herein is similar to the one found in Groh and Weaver (2015b) for flat beams.

As illustrated in Fig. 3, the beam is loaded with prescribed shear and normal tractions $\hat{T}_t$ in the hoop φ-direction and $\hat{P}_t$ in the radial r-direction on the top surface, and $\hat{T}_b$ and $\hat{P}_b$ on the bottom surface. These prescribed tractions can vary with φ. Additionally the beam may be loaded by a hoop stress or hoop displacement, and a transverse shear stress or transverse displacement at the two ends φ_A and φ_B of the beam. The presented model is an equivalent single layer model and hence the boundary conditions at the ends of the beam cannot be imposed layer-wise. Therefore, all the applied traction or displacement boundary conditions act through the whole beam thickness.

Fig. 4 shows the convention of naming the layers. The positions of the layers are numbered starting with t_0 at the bottom at $z = -t/2$ and ending at the top with t_N at $z = t/2$. Layer k is limited by