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Effective properties of short-fiber composites with Gurtin-Murdoch model of interphase

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ABSTRACT

A mathematical model employing the concept of the energy-equivalent inhomogeneity combined with the method of conditional moments has been applied to analyze short-fiber composites. The fibers are parallel, randomly distributed, and their interphase is assumed to be adequately described by the Gurtin-Murdoch material surface model. The properties of the energy-equivalent fiber, incorporating properties of the original fiber and its interphase, are determined on the basis of Hill's energy equivalence principle assuming its cylindrical shape. To describe random distribution of fibers a statistical method, the method of conditional moments, has been employed. Closed-form formulas for the components of the effective stiffness tensor of short-fiber reinforced composites have been developed which, in the limit, compare well with the results available in the literature for infinite parallel fibers with Gurtin-Murdoch interphase model. Influence of fiber length on contribution of the surface effects to the effective properties of the material containing cylindrical cavities has been analyzed for all five independent components of its stiffness tensor, and for two sets of surface properties in the Gurtin-Murdoch model.

1. Introduction

Technological advances in the area of composites with nano-size constituents have been paralleled by an increased interest in the effects of interphases. At nano-scale level, the ratio of the interphase volume to the volume of the material is higher than in more traditional materials and the radii of curvature of nano-components are smaller, both of which make the effects of interphases more pronounced and more important.

There are various features of the interphases that may determinate their influence on the overall properties of the composite materials. They include the difference between their properties and the properties of the constituents of the composite, their thickness in relation to the dimensions of those constituents, existence of the deformation-independent (residual) stresses that often exist when dissimilar materials are joined together, and various possible flaws, such as partial debonding.

One model capturing effects important in nano-composites is the material surface model of Gurtin and Murdoch (1975, 1978), (GM) (see also Benveniste and Miloch 2001). It assumes that the thickness of the interphase is negligible and that the resulting surface possesses its own material properties and residual stresses. The model has been widely used in analysis of local effects in nano-composites (Mogilevskaya et al., 2008; 2010) as well as in analysis of their overall properties (see Chen and Dvorak 2006; Chen et al., 2007; Duan et al., 2007a, 2007b; among others). In the later contributions the GM model is employed to analyze the effective properties of random nano-composites containing infinite cylindrical fibers of nano-size diameter.

All existing applications of the GM model were restricted to inhomogeneities consisting of spheres or infinite circular cylinders (affording two dimensional analysis). While that covers a number of practically important cases, composites containing inhomogeneities of other shapes do exist and their significance is likely to grow. For example, carbon nanotubes reinforced materials (CNTRM), currently under a rather intense development (Tserpes and Sylvester 2014), contain inhomogeneities in the form of fibers of various length. Their shape is better approximated by cylinders or prolate spheroids with high aspect ratio. In addition, previous studies of composites without interphases have shown that the results based on those two geometric approximations are very close (Kari et al., 2007). In the effort to obtain closed-form solutions for the effective properties of the composites, this closeness is exploited inhere and different geometric approximations are used in different aspects of analysis. So, extending application of the GM model to those (and other) shapes seems to be a timely endeavor, and it is precisely the topic of this work.

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Analysis of heterogeneous materials including GM model of interphases and inhomogeneities of shapes other than spherical is complex independently of the method employed. In the case of spheroidal or cylindrical inhomogeneities of finite length involving interphases, which are of immense practical significance (cf. Tserpes and Sylvester 2014), the numerical methods such as finite element or boundary element methods are, arguably, the only currently available methods capable of handling the problem. And even those methods are faced with very serious limitations (cf. Ma and Kim 2011; Tserpes and Sylvester 2014). Somewhat more versatile, albeit approximate, is a recently presented approach based on the concept of the energy-equivalent inhomogeneity. It is built on the premise that a particle of any shape and its (thin) interphase with the matrix can be equivalently replaced by a single uniform inhomogeneity with suitably adjusted properties that is then perfectly bonded with the matrix. The properties of the equivalent inhomogeneity can be found using energy approach (Hill 1963), formulated considering specifics of the employed interphase model. Introduced in its general form by Nazarenko et al., (2015), Nazarenko and Stolarski (2016) the concept of the energy-equivalent inhomogeneity has so far been applied only to spherical inhomogeneities. However, that was done exclusively to prove the concept, as the existing solutions used for comparisons and validations were available only for inhomogeneities of that shape.

In this work the energy-equivalent inhomogeneity approach is applied to short fibers modeled as cylindrical inhomogeneities of finite length with GM model of interphases. In comparison with the prolate spheroid geometry of short fibers this geometric approximation significantly simplifies development of their equivalent properties. It is subsequently combined with the method of conditional moments (MCM) to determine the effective properties for composites containing unidirectional randomly distributed short fibers. Here, in turn, approximation of short fibers as prolate spheroids simplifies the analysis. This combination facilitates development of closed-form expressions defining all components of the effective stiffness tensor of the considered composite. To the authors' best knowledge this is the first attempt to solve a problem of that type. Thus, to validate the proposed approach, the overall properties of unidirectional fibrous materials (i.e. containing infinite fibers) are obtained as a limiting case and compared with two-dimensional solutions of the problem, which are the only currently available results for cylindrical inhomogeneities with GM surface. In addition, relevance of the results presented in this paper to analysis of composites with various other orientations of short fibers is discussed to demonstrate versatility of the proposed approach.

With the above goals in mind the next section of the work briefly introduces the notion of energy equivalence and its subsequent specification for cylinders of finite length and GM model of interphases; properties of the equivalent inhomogeneity are defined. In Section 3 the basics of the MCM are outlined and the closed-form expressions for the effective properties of the unidirectional short-fiber composites are developed. This is followed by the numerical results and discussion, presented in Section 4. The paper final section, Section 5, contains some overall comments about the approach pursued herein and the results obtained. Several technical details are referred to Appendix A.

2. Energy-equivalent short-fiber with Gurtin-Murdoch surface model

2.1. General considerations

To find properties of the equivalent inhomogeneity of any shape, meant to incorporate properties of the original inhomogeneity and those of its interphase, the system is subjected to boundary displacements consistent with constant straining, repre-

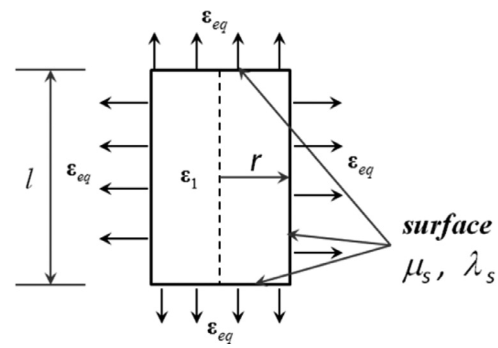

Fig. 1. Schematic illustration of cylindrical inhomogeneity.

sented by an arbitrary constant tensor $\boldsymbol{\varepsilon}_{\text{eq}}$. The elastic energy of this system is

$$E = \frac{1}{2}\int_{V_1} \varepsilon_1 : \mathbf{C}_1 : \varepsilon_1 dV_1 + E_{\text{int}}, \tag{2.1}$$

where E_{int} is the strain energy of the interphase, appropriate for the GM model, $\boldsymbol{\varepsilon}_1$ is the strain within the original inhomogeneity caused by $\boldsymbol{\varepsilon}_{\text{eq}}$ and $\mathbf{C}_1$ is the rank four tensor of elastic moduli of the original cylindrical inhomogeneity (Fig. 1).

The mathematical description of energy equivalence is expressed by the following equation

$$E = \frac{1}{2}\int_{V_{\text{eq}}} \varepsilon_{\text{eq}} : \mathbf{C}_{\text{eq}} : \varepsilon_{\text{eq}} dV_{\text{eq}} = \frac{1}{2}V_{\text{eq}}\varepsilon_{\text{eq}} : \mathbf{C}_{\text{eq}} : \varepsilon_{\text{eq}}$$
$$= E_{\text{int}} + \frac{1}{2}\int_{V_1} \varepsilon_1 : \mathbf{C}_1 : \varepsilon_1 \, dV_1, \tag{2.2}$$

where $\mathbf{C}_{\text{eq}}$ is the unknown constitutive tensors of the equivalent inhomogeneity and E_{int} depends on the specific model of the interphase employed and on the data characterizing the system. Under the assumption of linearly elastic interphase, at equilibrium both terms on the far right hand side are quadratic functions of $\boldsymbol{\varepsilon}_{\text{eq}}$ and Eq. (2.2) can be used to determine $\mathbf{C}_{\text{eq}}$. As shown in Nazarenko et al., (2015), Nazarenko and Stolarski (2016) that simple idea may be technically quite demanding, particularly for complex shapes of the inhomogeneity, but it is executable and in the cases considered so far leads to remarkably accurate, closed-form results.

2.2. Gurtin-Murdoch surface model and associated elastic energy

In the GM model the interphase is a surface assumed to be (linearly) elastic and to possess its own distinct properties as well as its own residual stresses. Thus, in this case, the equivalent inhomogeneity replaces the system consisting of the original inhomogeneity and its elastic surface. As explained in the introduction, at this stage of the analysis a short-fiber is assumed to be a cylindrical inhomogeneity whose dimensions and the relevant Lamé parameters are shown in Fig. 1.

Given that in the GM model of the interphase displacements are continuous (coherent), it follows that $\varepsilon_1 = \varepsilon_{\text{eq}}$ and – in the linear case considered in this work – the energy of the system is (see details in Nazarenko et al., 2015, Nazarenko and Stolarski 2016)

$$\mathbf{E} = \frac{1}{2}V_1(\varepsilon_1 : \mathbf{C}_1 : \varepsilon_1) + \frac{1}{2}\oint_{S_{\text{int}}} [\varepsilon_S : \mathbf{C}_S : \varepsilon_S + \tau_0 \nabla_S \mathbf{u} : \nabla_S \mathbf{u}] \, dS_{\text{int}}. \tag{2.3}$$

In the above equation $\nabla_S \mathbf{u}$ is the surface gradient of displacements, $\mathbf{C}_S$ is the isotropic tensor of surface elasticity

$$\mathbf{C}_S = 2\bar{\mu}_S \overset{4}{\mathbf{I}}_S + \bar{\lambda}_S \overset{2}{\mathbf{I}}_S \otimes \overset{2}{\mathbf{I}}_S, \tag{2.4}$$