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Analysis of an arbitrarily shaped interface cracks in a three-dimensional isotropic thermoelastic bi-material. Part 1: Theoretical solution

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ABSTRACT

The displacement and temperature discontinuity boundary integral-differential equation method is developed for the analysis of interfacial cracks in a three-dimensional isotropic thermoelastic bimaterial. The fundamental solutions for a unit point temperature and elastic displacement discontinuities on the interface are derived, and the explicit expressions for the temperature and displacements and stresses are derived. The hyper-singular boundary integral-differential equations of displacement and temperature discontinuities are obtained for a planar interfacial crack of an arbitrary shape. By virtue of the obtained boundary integral-differential equations and the method proposed for interface cracks in purely elastic media, the singular indices and the singular behaviors of the near crack-tip fields are studied. Meanwhile, the stress and heat flux intensity factors as well as the energy release rate are derived in terms of displacement and temperature discontinuities.

1. Introduction

Due to the wide use of composite materials like copper aluminium alloys in both mechanical and thermal environments, the study on the fracture behavior of interfacial cracks in thermal-elastic bimaterials is of great importance in engineering and has attracted a great deal of attention. As the strength of the bimaterials is significantly affected by the existence of interface cracks, the researches into the fracture behavior is of great importance. Sih (1962) pointed out that in homogeneous elastic media, the thermal stress near the crack tip has the classical singularity $r^{-1/2}$, which is identical to that for mechanical stresses. Chen and Ting (1985) pointed out that the temperature is proportional to $r^{1/2}$ while temperature gradient and stresses to $r^{-1/2}$ near the crack tip for an insulated crack. Owing to the discontinuities of the material properties and geometries, interfacial cracks have complex stress intensity factors with an oscillatory singularity $r^{-1/2+\varepsilon}$ with ε as the bi-material constant even under uniaxial loadings, which makes the problem more complex.

With regard to 2D problems, Brown and Erdogan (1968) initially studied an insulated Griffith interfacial crack under uniform heat flow and obtained the stress fields. Herrmann et al. (1979) compared the experimental results with numerical results based on finite element method of thermal cracking in dissimilar materials. Herrmann and Grebner (1982) studied a curved thermal crack in a brittle two-phase compound material and obtained the stress field from the closed solution of a boundary value problem. Martin-Moran et al. (1983) and Barber and Comninou (1983) respectively studied a penny-shaped interface crack subjected to a heat flow with perfect and imperfect contact and compared the difference between the two contact conditions. Later, Takakuda et al. (1984) used the complex function method to solve an external interface crack subjected to uniform temperature change or heat flow and obtained the distributions of displacements and stresses on the interface. Yuuki and Cho (1989) have proposed an extrapolation method to calculate the stress intensity factors for an interface crack. Lee and Shul (1991) analyzed a two-dimensional interface crack in an infinite plane under a far-field heat flow by virtue of the complex variable approach and obtained the thermal stress intensity factors. Munz and Yang (1992) derived the stress intensity factors of two bonded rectangular blocks of different materials

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subjected to thermal loading and pointed out that the stress distribution is dependent on the bonded component size. Chao and Chang (1994) analyzed the interface crack in dissimilar anisotropic media subjected to a remote heat flux and traction and derived the full-field closed form solutions for the displacements and stresses. Lee and Park (1995) studied a partially insulated interfacial crack subjected to vertically uniform heat flow in infinite, bonded dissimilar materials by deriving the Hilbert problem. Wilson and Meguid (1995) adopted thermal *J*-integral along with the mode separation concept to calculate thermal stress intensity factors. Sun and Quin (1997) proposed the modified crack closure integral along with displacement ratio for promoting accuracy in computing stress intensity factors. Martynyak (1999) studied a closed interface crack under compressive forces normal to its surface and a heat flow assuming the crack with imperfect thermal contact, and pointed out that the contact thermal resistance would influence the crack opening. Zhang (2000) analyzed an interfacial crack between two elastic layers and obtained the closed-form thermal energy release rate and stress intensity factors and indicated the key influence of the thermal stress intensity. Ikeda and Sun (2001) modified the virtual crack extension method as well as the modified crack closure integral method for solving thermal problem with the superposition method to obtain thermal stress intensity factors. Herrmann and Loboda (2001) utilized the contact-zone model to study an interface crack between two anisotropic spaces under remote mechanical-thermal loadings utilizing the Lekhnitskii-Eshelby-Stroh representation and obtained the induced temperature and displacement jump, stresses, and temperature flux. Kharun and Loboda (2004a, b) utilized the complex-function method to study a contact-zone interfacial crack model in isotropic and anisotropic bimaterials, respectively. This contact-zone model is also adopted by Herrmann et al. (2004) to study a thermally insulated interface crack under combined tension-shear mechanical loading and a temperature flux. Ratnesh and Chandra (2008) utilized the weight function method to analyze a 2D interface and found that the general expression for the interfacial crack is the same type as the homogeneous one. Pant et al. (2010) extended the element free Galerkin method and employed jump function technique to solve interfacial crack in bi-materials. Khandelwal and Chandra Kishen (2011) utilized body analogy method to analyze an interfacial crack subjected to thermal loads and obtained the analytical solution by computing the thermal weight function, with which the stress intensity factors are computed as well. Ma et al. (2013) studied a Zener–Stroh model of interfacial crack subjected to a uniform temperature shift and provided the interface defect tolerant size which can be used to assess the interface integrity and reliability under thermal load.

For 3D cases, Bregman and Kassir (1974) utilized the Muskhelishvili's method to study a penny-shaped interfacial crack subjected to uniform heat flow and got the stress intensity factors and energy release rate. Andrzej and Stanislaw (2003) used the potential theory method to study a plane crack lying on an interface in a microperiodic two-layered composite material under a uniform vertical heat flow. Johnson and Qu (2007) extended the interaction integral method to analyze curvilinear cracks in bimaterial interface under non-uniform temperature and obtained the induced stress intensity factors. Nomura et al. (2010) developed a numerical method using a path-independent H-integral to analyze the singular stress field of a 3D interface corner between anisotropic bimaterials subjected to thermal stress. Guo et al. (2012) investigated a plane crack problem of inhomogeneous materials with interfaces subjected to thermal loading using a modified interaction energy integral method and obtained the mixed-mode thermal stress intensity factors. Li et al. (2013) also used the universal weight function method to study a 3D interfacial crack in a bi-material under mechanical-thermal loading.

The displacement discontinuity boundary integral equation method is very efficient in solving crack problems as it grasps the intrinsic characteristic for crack problems as physical fields are discontinuous across crack faces. This method is also extended to solve interfacial crack problems in elastic media (Tang et al., 1998; Chen et al., 1999a, b), piezoelectric media (Zhao et al., 2004) and magnetoelectroelastic media (Zhao et al., 2008; Zhao et al., 2015).

Motivated by the current state of study of interfacial cracks, we develop the displacement and temperature discontinuity boundary hyper-singular integral-differential equation method for interfacial cracks in dissimilar, isotropic thermal elastic bi-materials.

The paper is organized as follows: Section 2 presents the basic equations, Section 3 derives the unit point displacement and temperature discontinuity fundamental solutions, and Section 4 derives the boundary hyper-singular integral-differential equations for an arbitrarily shaped interfacial crack. In Section 5, the singular behavior near the interfacial crack front is analyzed and the singular stress and heat flux fields ahead of the crack front are obtained in Section 6. The corresponding stress intensity factors and energy release rate are derived in Section 7, and conclusions are drawn in Section 8.

2. Basic equations

In the absence of body forces, the governing equations for a three-dimensional homogeneous thermal elastic medium in a steady state are (Shail, 1964)

$$\sigma_{ij,j} = 0, \tag{1a}$$

$$h_{i,i} = 0, \tag{1b}$$

where σ_{ij} and h_i are respectively the stress and heat flux, $i, j = 1, 2, 3$or $i, j = x, y, z$, and the index "i" or "j" after the comma denotes differentiation with respect to the coordinate.

In the Cartesian coordinates (x, y, z) and cylindrical coordinates (r, ϕ, z), the constitutive equations are expressed, respectively, in the form

$$\begin{aligned}
&\sigma_x = 2\mu\frac{\partial u}{\partial x} + \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \frac{E}{1-2\nu}\alpha\theta,\\
&\sigma_y = 2\mu\frac{\partial v}{\partial y} + \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \frac{E}{1-2\nu}\alpha\theta,\\
&\sigma_z = 2\mu\frac{\partial w}{\partial z} + \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - \frac{E}{1-2\nu}\alpha\theta, \qquad (2a)\\
&\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tau_{yz} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right), \tau_{zx} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right),\\
&h_x = -\beta\frac{\partial \theta}{\partial x}, \quad h_y = -\beta\frac{\partial \theta}{\partial y}, \quad h_z = -\beta\frac{\partial \theta}{\partial z},
\end{aligned}$$

$$\begin{aligned}
&\sigma_r = (\lambda + 2\mu)\frac{\partial u_r}{\partial r} + \lambda\left(\frac{1}{r}\frac{\partial u_\phi}{\partial \phi} + \frac{u_r}{r}\right) + \lambda\frac{\partial w}{\partial z} - \frac{E}{1-2\nu}\alpha\theta,\\
&\sigma_\phi = \lambda\frac{\partial u_r}{\partial r} + (\lambda + 2\mu)\left(\frac{1}{r}\frac{\partial u_\phi}{\partial \phi} + \frac{u_r}{r}\right) + \lambda\frac{\partial w}{\partial z} - \frac{E}{1-2\nu}\alpha\theta,\\
&\sigma_z = \lambda\frac{\partial u_r}{\partial r} + (\lambda + 2\mu)\frac{\partial w}{\partial z} + \lambda\left(\frac{1}{r}\frac{\partial u_\phi}{\partial \phi} + \frac{u_r}{r}\right) - \frac{E}{1-2\nu}\alpha\theta, \qquad (2b)
\end{aligned}$$