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Stress solution for functionally graded adhesive joints

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The present work provides a new model for the stress analysis of functionally graded adhesive lap joints with composite adherends. It is applicable to various joint configurations such as single lap joints, L-joints, T-joints, reinforcement patches, corner joints or balanced double lap joints. The modelling approach follows the concept of general sandwich-type analyses that consider only the overlap region of the joint with any combination of section forces and moments. To take into account shear deformations of the adherends, First Order Shear Deformation Theory is employed and bending-extension coupling of laminated adherends is covered. Several adhesive joint designs with various adhesive Young's modulus variations are investigated and the obtained adhesive stress distributions are compared to results of detailed finite element analyses. In general, a very good agreement is observed. The present model accurately renders that the adhesive peak stresses can be significantly reduced by the functional grading of the adhesive. Further, the effects of different functional grading functions of the adhesive Young's modulus on the adhesive stresses are studied and discussed. The paper concludes with design considerations for functionally graded adhesive joints.

1. Introduction

For an adequate design of adhesive lap joints the stresses in the bondline and the relevant design parameters as well as their effects on the mechanical behaviour of the joint have to be known to the engineer. Thus, the localized adhesive stress peaks at the end of the overlap, where failure is prone to emanate, are of special importance for a proper dimensioning of the joint. A great effort in research has been made in order to reduce the occuring stress concentrations by employing geometrical modifications of the joint for a more efficient design. A reduction of stress concentrations can, for instance, be obtained by using tapered adherends, rounded adherend corners or by modifications of the joint end geometry with spew fillets. A comprehensive overview of expedient geometrical modifications of adhesive lap joints and their effects on the stresses and effective joint strengths can be found in da Silva et al. (2011). However, with the proposed approaches the complexity of the joint geometry increases which is often unfavorable for industrial applications.

Another technique to reduce stress concentrations that does not require a geometrical change of the joint is motivated by the concept of functionally graded materials (FGM). FGMs are characterized by steadily varying material properties. FGMs were originally developed for super heat resistant materials for space aircrafts or nuclear fusion reactors (Koizumi, 1995) but are now of interest to researchers from many different disciplines. A comprehensive overview on FGMs can, for instance, be found in Miyamoto et al. (1999) or Jha et al. (2013). The concept of FGMs has been applied to adhesively bonded joints in various ways. Ganesh and Choo (2002) as well as Boss et al. (2003) achieved a spatial grading of the adherend's elastic modulus of a composite single lap joint employing a braided preform with varying braid angle. By means of finite element analyses, it was found that the peak shear stresses reduce by about 20%. They concluded that enhanced joint performance can be expected, but unfortunately the studies are not accompanied by experimental results. Apalak and Gunes (2007) studied adhesively bonded single lap joints with adherends composed of a functional gradient layer between a pure ceramic and a pure metal layer using finite elements. Their study showed that grading the adherend with respect to the thickness has no major impact on the adhesive stresses or strength. Many more researchers focused on functionally graded adhesives along the overlap region. Numerous works have addressed dual adhesive systems often referred to as bi-adhesives or mixed adhesives which can be seen as a stepwise functional grading of the adhesive. The idea of mixed adhesives has first been proposed by Raphael (1966) in order to reduce stress concentrations in adhesive joints with brittle adhesives by including a ductile adhesive at the ends of the overlap.

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However, in the last decade many researchers have intensively studied mixed adhesive joints with diverse results. Pires et al. (2003) as well as Fitton and Broughton (2005) studied adhesive joints with CFRP, steel and aluminium adherends theoretically using finite elements as well as experimentally and found that using bi-adhesives can reduce stress concentrations and increase joint strengths. However, some joint configurations did not benefit from mixed adhesives. Marques and da Silva (2008) and da Silva and Lopes (2009a) have found that dual adhesive systems improve the effective joint strength with respect to brittle adhesives in all cases but did not always out-perform joints with ductile adhesives.

More recently, functionally graded adhesives with mechanical properties that vary continuously along the overlap have been investigated and have proven to be successful. Only few experimental studies exist for such functionally graded adhesives due to the difficulties in achieving a smooth functional variation of the mechanical properties along the overlap in practice. Sancaktar and Kumar (2000) employed rubber particles to locally alter the adhesive whereas Stapleton et al. (2012) and Stapleton and Waas (2012) placed glass beads within the adhesive layer to obtain a varying density and stiffness along the overlap. In their studies, it was shown that employing a functionally graded adhesive yields a significant reduction of the peel stresses and an increase of the effective joint strength. Moreover, Stapleton et al. (2012) and Stapleton and Waas (2012) provided a joint finite element for functionally graded adhesive joints with an embedded analytical solution in order to allow for an efficient finite element analysis for an array of joint types and studied whether multiple loading conditions significantly impact the effectiveness of the functional grading. More recently, Carbas et al. (2014a) implemented functionally graded adhesive joints using induction heating giving a graded cure of the adhesive over the overlap length. It was found that functionally graded adhesive joints have a significantly higher joint strength compared to joints with uniformly cured adhesives.

Only a limited number of analytical models for the analysis of functionally graded adhesive joints are available in literature. Such analytical models that allow for a fast analysis of the load transfer are particularly important for parametric studies in early design stages and for optimization processes. Raphael (1966) was the first who proposed an analytical model for mixed adhesive joints based on a simple shear-lag model as proposed by Volkersen (1938). Srinivas (1975) developed a model for mixed adhesive lap joints with dissimilar adherends as well as composite adherends with the drawback of requiring an additional numerical solution. das Neves et al. (2009a; 2009b) derived a model for mixed adhesive single and double lap joints based on the analysis of Frostig et al. (1999). The first analysis model considering a smooth and continuous grading of the adhesive has been proposed by Kumar (2009). The derived model for axially loaded adhesively bonded tubular lap joints with similar as well as dissimilar adherends is based on a variational principle which minimizes the complementary energy of the joint. Exponential as well as polynomial variations of the adhesive Young's modulus along the overlap are investigated and it is found that the peak adhesive stresses are reduced in all cases. Based on this model several refined approaches for cylindrical functionally graded adhesive joints have been proposed (Kumar and Mittal, 2013; Kumar and Scanlan, 2010; 2013). Spaggiari and Dragoni (2014) proposed a model for the analysis of cylindrical functionally graded adhesive lap joints under torsion and analyzed the optimal distribution of the adhesive stiffness to achieve a constant adhesive shear stress along the overlap. Carbas et al. (2014b) derived a closed-form analytical formula for the adhesive shear stresses for flat functionally graded adhesive single lap joints under tension based on the model by Volkersen (1938) and validated the model with finite element results. Stein et al. (2016) developed an analysis model for planar axially loaded single lap joints with a functionally graded adhesive based on the same kinematical assumptions as the analytical model proposed by Goland and Reissner (1944). In their study, the effects of a quadratic, biquadratic and cosine distribution of the adhesive Young's modulus on the adhesive stresses have been studied and validated with numerical results of detailed finite element analyses. However, these approaches are joint specific and limited in their conditions of applicability.

For the case of adhesive lap joints with a single isotropic adhesive, a generalized modelling approach that overcomes the limited applicability of the classical models exists. In this approach, often referred to as general sandwich-type analysis, only the overlap region of the adhesive joint under general loading is considered to allow for the analysis of various joint configurations. The earliest general sandwich-type analysis is the one proposed by Bigwood and Crocombe (1989). They modelled the adherends as Euler–Bernoulli beams and the adhesive layer as smeared springs. Many extensions of their model regarding non-linear material behaviour of the adhesive and adherends (Bigwood and Crocombe (1990); 1992)) or the influence of environmental degradation (Crocombe (2008)) have been proposed. Further, the First Order Shear Deformation Theory (FSDT) has been employed in general sandwich-type analyses, e.g. by Liu et al. (2014) and Weißgraeber et al. (2014) to take into account the adherends' shear deformations as they are important for laminated adherends. The latter also considered asymmetrically laminated adherends taking into account bending-extension coupling and used a more complete kinematical approach for the adhesive layer.

The aim of the present work is to provide an efficient general sandwich-type model for functionally graded adhesive joints that allows for the analysis of various joint configurations with composite adherends including bending-extension coupling.

2. Governing equations

The proposed analysis approach generalizes the above mentioned models for functionally graded adhesive joints by considering only the overlap region of the adhesive joint with arbitrary section forces and moments. In this way, the model allows for the analysis of a wide range of adhesively bonded joint configurations featuring a single overlap, such as single lap joints, L-joints, T-joints, peel joints, reinforcement patches or corner joints, cf. Fig. 1.

Additionally, as a special case, balanced double lap joints can be modelled. The symmetry of the joint can be considered investigating only half of the middle layer and suppressing the corresponding shear and bending deformations. More details are given in Section 3. Only the corresponding end loads acting on the overlap region have to be determined for the application of the general sandwich-type model. The considered two-dimensional sandwich-type element of the overlap with normal force N, shear force V and bending moment M at each end of the top and bottom adherend is illustrated in Fig. 2.

The index 1 is used for parameters belonging to the upper adherend, the index 2 for the lower adherend and the index a for the adhesive. Hence, the respective adherend thicknesses for the top and bottom adherend are denoted by h_1 and h_2 and the adhesive thickness is denoted by t. The width of the sandwich-type element is b and the overlap length is L.

In most closed-form analytical models for adhesive joints, a system of differential equations in terms of adhesive stresses with corresponding boundary conditions is derived by repeated differentiation and substitution of the main field equations, i.e. equilibrium equations, constitutive equations and kinematics. In the case of functionally graded adhesive joints the adhesive Young's and shear modulus depend on the horizontal coordinate x which has to be taken into account properly. Due to the fact that the order