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# On buckling of a soft incompressible electroactive hollow cylinder

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ABSTRACT

Soft electroactive materials show great potential for device and robot applications. However, these materials are apt to experience buckling and pull-in instability under critical pressure or voltage, and, therefore, their practical applications are more or less prevented. In this paper, buckling behavior of incompressible soft electroactive hollow cylinders is investigated based on the nonlinear theory of electroelasticity and the associated linear incremental field theory. Hollow cylinders including or excluding the effects of exterior electric field are studied in a comparison manner. The equations governing the linearized incremental motion upon a finitely deformed configuration in the presence of an electric field are derived and exactly solved by introducing three displacement functions. As an illustrative example, the generic isotropic electroactive materials are considered and results are presented for a simple model of ideal electroelastic material. Numerical calculations show that the buckling of electroactive hollow cylinders is significantly influenced by the biasing fields, the electromechanical coupling parameters, the geometrical parameters of the cylinder, and the electric field outside the cylinder. In particular, a phase diagram is constructed based on the numerical results to clearly identify the dominant buckling modes and the transition between them in the $\kappa - -\nu$ (axial wave number versus radius ratio) plane.



## 1. Introduction

Commonly, the instability phenomenon is considered as negative that should be accurately predicted and carefully avoided. Stabilities of elastic bodies have attracted considerable attention since Euler's classical work on the buckling of thin columns (Euler and Carathéodory, 1952). It is well known that a slender elastic body will buckle under a sufficiently large compressive load, the so-called Euler buckling.

Wilkes (1955) seems to be the first to address the instability of a cylindrical shell under axial load by using three-dimensional theory of nonlinear elasticity. The linearized system around a finite axial strain was solved exactly in terms of Bessel functions. It was found that the bifurcation curves for the incompressible neo-Hookean material have a horizontal asymptote of $\lambda \approx 0.444$, which corresponds to surface instability of a compressed half-space. However, Wilkes's calculation was confined to the axisymmetric mode ($n = 0$). Pan and Beatty (1997) considered an incompressible cylindrical tube and derived the formulation of bifurcation criterion including both axisymmetric mode ($n = 0$) and asymmetric mode ($n = 1$). They compared the results obtained using neo-Hookean, Mooney–Rivlin and Gent–Thomas models. Dorfmann and Haughton (2006) extended the theory to look at a selection of compressible materials and considered higher order bifurcation modes. It was shown that long thick-walled tube undergoes asymmetric mode buckling while short thick-walled tube undergoes axisymmetric mode buckling at some critical loading. However, for thin-walled tube, the occurrence of various critical bifurcation modes ($n = 0, 1, 2, 3, \ldots$) depends on the length of the tube. Goriely et al. (2008) used the Stroh formalism to investigate the instability of an incompressible cylindrical shell under axial load, and expanded the exact solution up to order 2, 4 and 6 so as to make a comparative study.

Soft electroactive materials have appeared as smart materials which enable realizing the conversion between electrical and mechanical energies. Due to their low weight, rapid response and large deformation under electrical stimulus, soft electroactive materials are widely used to develop high-performance devices such as actuators, artificial muscles, energy harvesters and space robotics (Pelrine et al., 2000; Bhattacharya et al., 2004; O' Halloran et al., 2008). The strong nonlinearity and the electrome-

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chanical coupling should be taken into account in the analysis of soft electroactive materials or structures. The formulation of a general nonlinear theory of electroelasticity was first developed by Toupin (1956) and has been intensively studied in recent years (McMeeking and Landis, 2005; Dorfmann and Ogden, 2006; Ericksen, 2007; Suo et al., 2008; Ogden, 2009).

Electroactive materials are often working under extremely high voltage, thus susceptible to fail in the form of electromechanical instability which can be a precursor of electrical breakdown (Stark and Garton, 1955; Blok and LeGrand, 1969). Such instability phenomenon poses a clear limitation in the development of devices based on electroactive materials. Careful modeling and analyses are therefore required.

Experiments showed the coexistence of two regions, i.e., flat and wrinkles, in an electroactive film when subjected to critical applied voltage. Stable wrinkles arise first and then pull-in instability occurs as the applied voltage keeps increasing (Plante and Dubowsky, 2006). Zhao et al. (2007) and Zhao and Suo (2007) demonstrated that the electroactive film will undergo inhomogeneous deformation because the free-energy function of the elastomer is typically nonconvex, and that the electromechanical instability occurs when the generalized tangent modulus ceases to be positive definite. They further revealed that the electromechanical instability of electroactive materials can be considerably enhanced by prestress. Dorfmann and Ogden (2010) considered the stability of an incompressible electroactive half-space subjected to pure homogeneous compression in the presence of an applied electric field normal to its surface. They (Dorfmann and Ogden, 2014) also investigated the instability of an equibiaxially stretched electroelastic plate with and without the effects of exterior electric field, which exhibits different features because of the differences in electric boundary conditions. Díaz-Calleja et al. (2009) analyzed the bifurcation characteristics of two biaxially stretched incompressible rubber slabs, one with flexible electrodes on its two surfaces, and the other floating between two fixed electrodes. It was shown that the effects of electric field on the bifurcation of the two slabs are inverse and the pull-in phenomenon in the second case can be prevented if the gap between the sample and the electrodes is large enough. Recently, dielectric composites have attracted considerable interests because they can markedly improve the performance of dielectric devices (Huang et al., 2004; deBotton et al., 2007). The stability of multilayered soft dielectrics were systematically investigated by Bertoldi and Gei (2011) and Rudykh and deBotton (2011).

Dielectric elastomer tube actuator (DETA) was proposed first by Pelrine et al. (1998). Compared to other dielectric elastomer configurations (for example, plate and membrane actuators), DETA has very low inactive-to-active material ratio, and is less bulky but more versatile for applications (Carpi and Rossi, 2004; Cameron et al., 2008). Similarly, DETA is also susceptible to electromechanical instability. Thus, it is necessary to analyze the stability of electroactive material of cylindrical structure. Zhu et al. (2010) obtained an analytical solution for the finite deformation of a DETA which is mechanically prestretched and actuated by a voltage. They also studied the effects of prestretch and geometry on electromechanical instabilities of the tube actuator using a linear perturbation method. Zhou et al. (2014) demonstrated that the electromechanical instability of a DETA could be avoided, and larger actuation deformation may be achieved by applying specific boundary constraints. Most recently, Zhang et al. (2015) examined four failure modes of a DETA, including loss of tension, electrical breakdown, snap-through instability and tensile rupture.

The so-called Hessian approach was widely adopted to analyze the stability of electroactive material (Zhao et al., 2007; Zhao and Suo 2007; Díaz-Calleja et al., 2009). However, these analyses were only limited to homogeneous deformations, and the associate results did not reflect the dependence of buckling behavior on the geometry of the structure. Taking both geometry and non-homogeneous deformation into consideration, the occurrence of diffuse modes of an electroactive hollow cylinder subjected to a uniform biasing electric field is exactly investigated in this paper. The governing equations will be derived in form of incremental fields upon a finite deformation state. By introducing three displacement functions, exact solutions are presented in terms of Bessel functions, and, with the incorporation of boundary conditions, the characteristic equation for buckling behavior of the cylinder is derived. A simple prototype model, namely the neo-Hookean electroelastic material, is considered as an illustrative example. A simple scaling law involving the normalized critical compression and normalized wave number is derived for tuning the buckling behavior of the cylinder. Numerical calculations will be performed to examine the effects of material parameters, biasing electric field, and geometrical configuration on the buckling properties of the cylinder. The influence of the electric field in the vacuum exterior to the cylinder will also be investigated through comparative studies.

## 2. Finite deformation and incremental motion

For better understanding the derivations, the general formulations associated with the nonlinear electroelasticity and the linear incremental field theory (Dorfmann and Ogden, 2006; Ogden, 2009) are briefly reviewed in Appendix A. Consider an isotropic, incompressible soft electroactive hollow cylinder with initial length $L$, inner radius $R_i$, and outer radius $R_o$, respectively. The radius ratio of the cylindrical shell is denoted by $\nu = R_i/R_o$. The cylinder is assumed subjected to an axial mechanical load and a uniaxial biasing electric displacement $D_z$ that is aligned with the axial direction. The hollow cylinder then deforms to become a one with length $l$, and inner and outer radii $r_i$ and $r_o$, respectively. The initial and current cylindrical coordinates are respectively designated as $(R, \Theta, Z)$ and $(r, \theta, z)$ with the origin located at the center of one end of the cylinder. For incompressible materials, the principal stretches of the cylinder are

$$\lambda_z = \lambda, \quad \lambda_r = \lambda_\theta = \lambda^{-1/2} \tag{1}$$

Then, the deformation gradient $\mathbf{F}$ is given by

$$\mathbf{F} = \begin{bmatrix} \lambda^{-1/2} & 0 & 0 \\ 0 & \lambda^{-1/2} & 0 \\ 0 & 0 & \lambda \end{bmatrix}. \tag{2}$$

For axisymmetric deformation, the non-zero components of the total stress tensor and electric field vector may be derived from Eq. (A1) as (Dorfmann and Ogden, 2006; Ogden, 2009; Chen and Dai, 2012):

$$\tau_{rr} = \tau_{\theta\theta} = 2W_1\lambda^{-1} + 2W_2\left(I_1\lambda^{-1} - \lambda^{-2}\right) - p, \tag{3a}$$

$$\tau_{zz} = 2W_1\lambda^2 + 2W_2\left(I_1\lambda^2 - \lambda^4\right) + 2W_5 D_z^2 + 4W_6\lambda^2 D_z^2 - p, \tag{3b}$$

$$E_z = 2\left(W_4\lambda^{-2} + W_5 + W_6\lambda^2\right)D_z, \tag{3c}$$

where $W_m = \partial W/\partial I_m$, with $I_m$ being the $m$th invariant as given in Eq. (A2). It is straightforward from Eq. (3c) that

$$D_z = \frac{E_z}{2\left(W_4\lambda^{-2} + W_5 + W_6\lambda^2\right)} = \varepsilon E_z, \tag{4}$$

which implies that the permittivity of the material relies on the deformation of the cylinder, i.e., $\varepsilon = \varepsilon(\lambda)$, if $W_4 \neq 0$ or $W_6 \neq 0$.

Considering the effects of exterior electric fields ($r < r_i$ or $r > r_o$), the boundary conditions in the second and third equa-