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Modelling of micro-inertia effects in closed-cell foams with application to acoustic and shock wave propagation

R. Barthélémy[a,b], N. Jacques[a,*], S. Kerampran[a], F. Vermeersch[b]

[a] ENSTA Bretagne, FRE CNRS 3744, IRDL, 2 rue François Verny, F-29806 Brest cedex 9, France
[b] MBDA France, 1 avenue Réaumur, F-93350 Le Plessis-Robinson, France

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ABSTRACT

A continuum approach is proposed to describe micro-inertia effects in closed-cell foams using a micromechanical method. An initially spherical unit-cell was considered and the influence of inertia at the unit-cell level was characterised with the use of a dynamic homogenisation technique. The contribution of micro-inertia appears in the form of a dynamic component of the macroscopic stress. A closed-form expression of the dynamic stress was obtained. The proposed modelling was applied to acoustic and shock wave propagation. In both cases, the influence of micro-inertia was found to be significant. The obtained results are in good agreement with existing data of the literature, provided by micromechanically accurate finite element computations and experiments. The proposed model is aimed to enhance continuum models of foam materials by taking into account the contribution of micro-inertia.

1. Introduction

Foams and cellular materials present many attractive properties coupled with a lightweight structure. In particular, foams are known to have excellent energy absorbing capabilities and are widely used in the automotive, aeronautic and spatial industries for impact protection and shock mitigation applications (Schwingel et al., 2007; Banhart and Seeliger, 2008). Foams are highly porous media with a relative density (ratio of the density of the foam to the one of the base material) lower than 0.3 (Gibson and Ashby, 1997). Two main kinds of foam materials exist, and can be characterized by their distinct microstructures: open-cell foams are made of interconnected ligaments joining at vertices while the microstructure of closed-cell foams consists of closed cells separated by membrane-like walls (Fig. 1).

Several experimental investigations were devoted to the mechanical behaviour of metal foams under quasi-static and dynamic loading conditions, see e.g. (Deshpande et Fleck, 2000a, 2000b; Danneman et Lankford, 2000; Tan et al., 2005, 2012; Mukai et al., 2006; Elnasri et al., 2007). The compressive response of a foam specimen generally exhibits two phases: a plateau stage in which the stress remains almost constant, and a densification stage in which the stress increases strongly with strain (see Appendix D). This particular behaviour is responsible for the high energy absorbing capabilities of foams, but also promotes the formation of shock waves. When a foam specimen is impacted above a certain velocity, named critical velocity, a shock wave is generated at the impact surface. Several experimental studies have shown that critical velocities for cellular materials are much smaller than those for dense solids, of the order of some tens of m/s, see Table 1.

The behaviour of cellular materials under shock loading is generally different from the one observed in static compression, with enhanced crushing strength and energy absorption capabilities (Reid and Peng, 1997; Zhao et al., 2005, Zou et al., 2009). In a recent work, Barnes et al. (2014) were able, using impact tests and high-speed photography, to measure the velocity, stress and strain increments induced by the propagation of a shock wave in an open-cell aluminium foam. Their experimental technique allowed a direct experimental characterisation of the Hugoniot curve of the foam. They have observed that the Hugoniot is different from the quasi-static compressive response of the foam. For a given stress increment, a larger strain is achieved under shock compression.

The shock behaviour of cellular solids was also investigated using micromechanical numerical simulations, in which the geometry of the foam microstructure is explicitly represented. This approach was initiated by Zou et al. (2009) in the case of honeycombs. Recently, three-dimensional simulations of shock propagation in foams with a random microstructure were proposed. Gaitanaros and Kyriakides (2014) considered the case of open-cell foams. Through comparison with the experiments of Barnes et al. (2014), they have shown that these simulations are able to reproduce with a very good accuracy the Hugoniot response of foams. A similar study was performed by Zheng et al. (2014) for

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[*] Corresponding author. Fax: +33 2 98 34 87 30.
E-mail address: nicolas.jacques@ensta-bretagne.fr (N. Jacques).

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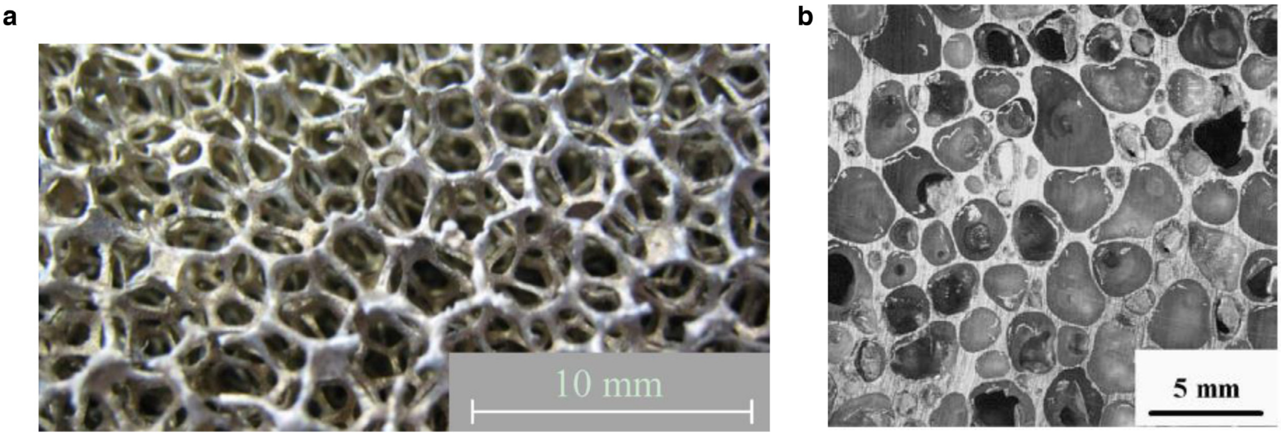

Fig. 1. Microstructure of two aluminium foams: (a) open-cell foam with a relative density of 0.06 and (b) closed-cell foam with a relative density of 0.16. Reproduced from (a) (Vesenjak et al., 2012) and (b) (Mukai et al., 2006).

Table 1
Critical impact velocities observed for different cellular materials.

Reference	Material	Relative density	Critical impact velocity
Tan et al. (2005)	Hydro/Cymat closed-cell aluminium foam	0.05	≈ 40 m/s
Tan et al. (2005)	Hydro/Cymat closed-cell aluminium foam	0.13	≈ 110 m/s
Elnasri et al. (2007)	Alporas closed-cell aluminium foam	0.09	< 55 m/s
Elnasri et al. (2007)	Ateca Nickel hollow spheres	0.025	< 55 m/s
Tan et al. (2012)	Duocel open-cell aluminium foam	0.1	≈ 110 m/s
Tan et al. (2012)	Duocel open-cell aluminium foam	0.088	≈ 70 m/s
Barnes et al. (2014)	Duocel open-cell aluminium foam	0.083	< 60 m/s

closed-cell foams. They also have found that the macroscopic stress-strain response of a foam under shock compaction is different from the quasi-static one. It is worth noticing that in the simulations of Barnes et al. (2014) and Zheng et al. (2014), the base material had a rate-independent elastic-plastic behaviour. Therefore, the influence of the loading rate on the macroscopic foam response is not a material rate effect, but is related to local dynamic effects induced by the rapid crushing of the foam cells. These effects are generally referred to as micro-inertia effects in the literature (Tan et al., 2002, 2005; Zhao et al., 2005). Micro-inertia effects are a natural outcome of three-dimensional micromechanical simulations (because the microstructure is represented explicitly). However, this kind of simulations is complex and requires high computing power. Therefore, it is highly desirable to develop continuum models for foam materials taking micro-inertia into account. Romero et al. (2008, 2010) have developed a multiscale formulation to predict the dynamic response of open-cell foams. In this approach, the effective (macroscopic) behaviour of the foam is obtained by considering a microscopic unit-cell. The response of the unit-cell is modelled by using a dynamic version of the principle of minimal action. To our best knowledge, a similar model does not exist for closed-cell foams.

In the present work, a continuum model of micro-inertia in closed-cell foams is proposed. The modelling is based on a multiscale approach and the use of the dynamic averaging technique introduced by Molinari and Mercier (2001). Within this framework, micro-inertia effects appear in the form of a dynamic component of the macroscopic stress. The proposed model is aimed to enhance continuum models of foam materials by taking into account the contribution of micro-inertia.

The paper is organised as follows. In Section 2.1, the dynamic homogenisation procedure introduced by Molinari and Mercier (2001) is reviewed and recast in a Lagrangian form. The dynamic stress for closed-cell foams is analytically derived in Section 2.2, by considering an initially spherical shell as a Representative Volume Element. In Section 3.1, an application of the present model to acoustic wave dispersion is presented. The influence of micro-inertia on shock wave propagation is discussed in Section 3.2. The obtained results are compared to existing data of the literature.

2. Micromechanical model for micro-inertia effects in closed-cell foams

2.1. Dynamic homogenisation framework

Micro-inertia effects in closed-cell foams have been characterised using the dynamic homogenisation technique proposed by Molinari and Mercier (2001). This method was initially developed to describe the dynamic behaviour of porous media and then applied to several problems of dynamic fracture of ductile solids by micro-voiding, see e.g. (Czarnota et al., 2008; Wright and Ramesh, 2008; Jacques et al., 2012; Molinari et al., 2014; Molinari et al., 2015). In the present section, a Lagrangian form of the Molinari-Mercier averaging procedure is proposed.

We consider a Representative Volume Element (RVE) of an heterogeneous solid. The domain occupied by the RVE in the initial state is denoted by Ω_0. Let x_i and x_j^0 $(i,j = 1,2,3)$ be the components in a Newtonian coordinate system of the current and initial positions of a given material point of the RVE. The microscopic deformation gradient $\partial x_i/\partial x_j^0$ is denoted by f_{ij}. Inertia effects are taken into account at the scale of the RVE. Therefore, the microscopic nominal stress tensor p_{ji} satisfies the equation of balance of linear momentum:

$$\frac{\partial p_{ji}}{\partial x_j^0} = \rho_0\, \gamma_i \tag{1}$$

where γ_i and ρ_0 are respectively the acceleration and the initial mass density of any material point in the RVE. We consider a virtual incremental displacement field δu_i. The dynamic principle of

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