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Exploiting measurement-based validation for a high-fidelity model of dynamic indentation of a hyperelastic material

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The high-speed (up to 1 m/s) loading and unloading of a rubber block by a rigid wedge has been simulated using a finite element model. The model has been refined and validated using measured data obtained from combined fringe projection and two-dimensional digital image correlation during the real-time loading and unloading of a rubber block by an aluminium wedge. Comprehensive data was obtained from the experiment in the spatial and temporal domains at resolutions of 0.075 mm and 0.00125 s respectively. Image decomposition techniques were used to represent the predicted and measured displacement fields in order to allow a number of quantitative measures to be employed to assess the fidelity of the model. It was demonstrated that it was essential to refine the Mooney–Rivlin material model to provide an accurate representation of the material behaviour for the speeds of the loading and unloading. The refined model was validated using the European Committee for Standardization (CEN) procedure and found to exhibit differences relative to the experiment of less than 5%. Together the predicted and measured data fields probably represent the best description yet of a soft material being deformed by a rigid indenter.

1. Introduction

Nowadays, the investigation of engineering problems that involves materials which experience large deformation, such as rubber and biological materials, has focused the attention of many researchers (Hamley, 2007). An important aspect of research on such materials is the analysis of their behaviour when subject to indentation for which it is important to be able to describe their mechanical behaviour using theoretical models. Analytical (Hertz, 1881; Barber and Ciavarella, 2000; Johnson, 1985; Malits, 2011; Dini et al., 2008; Truman et al., 1995; Ciavarella et al., 1998) and numerical (Jayadevan and Narasimhan, 1994; Jaffar, 2002; Jaffar, 2003) studies are available in the literature which all include significant assumptions. For instance, that strains are small and they are below the elastic limit (Malits, 2011; Truman et al., 1995; Jaffar, 2002), that an elastic half-plane can be considered (Dini et al., 2008; Ciavarella et al., 1998; Jaffar, 2002; Jaffar, 2003), that the contact area is much smaller than the radius of the indenter, that there is frictionless contact (Truman et al., 1995; Jaffar, 2002; Jaffar, 2003) or that the external angle of the indenter is very small (Ciavarella et al., 1998). Some of these assumptions are not always well supported by experiments (Xiao et al., 2014; Xiao et al., 2010; Burguete and Patterson, 1997; Han et al., 2012). Although data from experiments is essential for the validation of analytical and numerical models, no substantial work can be found in the literature relating to integrated analysis involving modelling and experiments of high-speed, large indentation of soft materials. In this study, a simple low-cost approach for the measurement of three-dimensional displacements is applied to the analysis of the high-speed indentation of a block of hyperelastic material by a rigid indenter and the resultant data is used to refine and validate a computational model of the event.

One of the most extensively used techniques for full-field three-dimensional displacement measurements is digital image correlation (Sutton et al., 2009), which for high speed events, as in case of this work, requires the use of two high-speed cameras. In this study, to avoid the need for two expensive high-speed cameras, an alternative technique has been used which combines two well-known methods namely two-dimensional digital image correlation (Pan et al., 2009) and fringe projection (Heredia-Ortiz and Patterson, 2003). Since the two methods employ similar experimental set-ups, i.e. a camera perpendicular to the measured surface, it is possible to measure simultaneously in-plane and out-of-plane displacements using only one camera and a fringe

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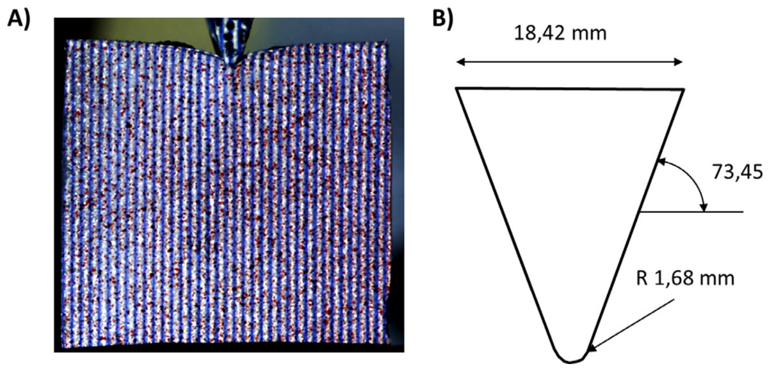

Fig. 1. A photograph (left) of the silicone block during contact by the indenter whose dimensions are given in the diagram (right).

projector (Felipe-Sesé et al., 2014a,b). Two-dimensional Digital Image Correlation (2D-DIC) has been used to measure in-plane displacements and Fringe Projection (FP) to measure out-of-plane displacements. The resultant data was used to refine a finite element model of the hyperelastic model undergoing high-speed indentation by the rigid wedge using innovative, quantitative comparison methods (Sebastian et al., 2013). An initial computational model was found to give relatively poor results and so additional material compression tests were performed at different load rates to refine the calibration of the material model. Tchebichef polynomials (of the first kind) (Sebastian et al., 2013) were used to decompose maps of surface strain from both experiment and simulation to permit detailed quantitative comparisons. The variation of the surface strain fields as a function of time increased the dimensionality of the comparison process compared to some prior work (Xiao et al., 2014; Sebastian et al., 2013) and hence a number of quantitative measures have been explored as means of evaluating the degree to which the model was a surrogate for the experiment and these are believed to represent an advance compared to earlier comparison work for dynamic cases (Burguete et al., 2014).

2. Numerical modelling

An explicit numerical analysis was conducted using the commercially-available software, Abaqus 6.11 (2011) for a silicone rubber block of dimensions 60 mm × 60 mm × 20 mm subject to indentation at speeds varying from 400 to 1000 mm/s by an aluminium wedge of thickness 20 mm with a tip radius of 1.68 mm and an included angle of 73.45° as shown in Fig. 1. The indenter was modelled as a non-deformable body using 520 bi-dimensional rigid solid elements (type R3D3). The unfilled silicone block was modelled using 150,000 four-noded reduced integration elements (type C3D8I). The size of these elements varied according to their position in the block, as shown in Fig. 2. The minimum element size controlled the minimum time-step in the explicit solution and so the values were chosen to give an appropriate temporal and spatial resolution. The time step Δt is related to the element size by the Courant–Friedrichs–Lewy (CFL) condition (Courant et al., 1967) which states that for stability the time step must be sufficiently small to ensure that information has time to propagate through the spatial discretization, i.e.

$$\Delta t \leq \Delta t_{cr} = min\left(\frac{L_{element}}{c_d}\right) \tag{1}$$

where $L_{element}$ is the minimun element length and c_d is the dilatational wave speed in the material, which assumed from the literature to be 1050 m/s (Tables of Physical, and Chemical Constants, 2005).

The silicone rubber is known to exhibit a non-linear elastic behaviour that can be modelled using the Mooney–Rivlin relationship (Mooney, 1940; Rivlin, 1948) in which the strain energy function is described by (Adkins and Rivlin, 1997)

$$U = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \tag{2}$$

where U is the strain energy per unit of a reference volume, I_1 and I_2 are the first and second stress invariants and C_{10} and C_{01} are the specific material parameters. In the first instance, the material parameters were taken as $C_{10} = 0.17$ and $C_{01} = 0.1$ based on the Shore A Hardness of the material (42 from experiments) and the Batterman and Köhler relationship (Battermannm and Köhler, 1982). Subsequently, the material model was refined by providing empirical data from compression tests which were used by the Abaqus subroutine to evaluate the response of the material. The uniaxial compression tests were performed on two cubes of the material with a side of length 30 mm which were loaded between platens at 400 m/s and 1000 m/s respectively using the same machine as described below for the indentation experiments. The results from these tests are shown in Fig. 3. The applied load was obtained from the load cell of the test machine and used to compute engineering stress by dividing the applied load by the original cross-section area of the specimen; while the displacement of the cross-head of the test machine was used to evaluate engineering strain by dividing the measured displacement by the original height of the specimen. The data in Fig. 3 were used to evaluate the material parameters, C_{10} and C_{01} using the EDIT MATERIAL tool from Abaqus 6.11 (2011), and were found to be 1.5 and 1.1 respectively at 400 m/s and 2.1 and 1.0 respectively at 1000 m/s.

The movement of the wedge into the block and the release of the wedge after indentation were both modelled using wedge speeds obtained from the experiments. The results from the initial and refined models are shown in Figs. 4 and 5.

3. Experiments

3.1. Measurement technique

Indentation and contact of soft materials, such as silicone rubber, usually generates large in-plane displacements and significant out-of-plane displacements. This implies the need to measure surface displacements in all three dimensions simultaneously during the dynamic loading. In this study a combined fringe projection and two-dimensional digital image correlation method has been utilised based on earlier work (Felipe-Sesé et al., 2014a, b). The method employs a conventional colour LCD projector and a colour RGB high-speed camera, as shown schematically in Fig. 6. The projector is used to create a blue & white fringe pattern with a sinusoidal intensity profile in one direction on the measured surface, as shown in Fig. 1, which permits fringe projection analysis to be performed in the blue spectrum. In addition, a red speckle pattern on a white background is painted onto the measured surface to