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Introducing the concept of directly connected rheological elements by reviewing rheological models at large strains

R. Kießling*, R. Landgraf, R. Scherzer, J. Ihlemann

Professorship of Solid Mechanics, Faculty of Mechanical Engineering, Technische Universität Chemnitz, Reichenhainer Straße 70, D-09126 Chemnitz, Germany

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ABSTRACT

Rheological models are often used to exemplify the structure of phenomenological material models. For this purpose, different elements, representing elastic, viscous or plastic material behaviour, are combined in parallel and series connections. This article introduces a concept to material modelling within the framework of multiplicative decomposition of the deformation gradient. Thereby, the basic idea of rheological connections is directly applied. Assuming the additive decomposition of the stress power, relations for parallel as well as series connections are derived and the thermodynamic consistency of the concept is proved. To exemplify the modelling concept, an existing viscoplastic model of overstress type, which is motivated by a rheological model, is reviewed and reproduced. To this end, specific nonlinear material models representing elastic, viscous and plastic material behaviour are applied in a thermodynamically consistent manner and the connection relations are evaluated. Finally, a numerical procedure is proposed, which implements the connection relations directly rather than solving the evolution equations for inelastic strains. Both the analytical derivation and the numerical procedure based on the connection relations yield equal results compared to standard methods, which highlights the applicability of the presented concept for the development of models of multiplicative inelasticity at large strains.

1. Introduction

Rheological elements and connections often represent the basis in the development of complex material models. According to this modelling concept, rheological elements, symbolising elastic, viscous or plastic material behaviour, are connected in such a way that inelastic material models can be deduced (Reiner, 1968). To this end, kinematic relations of the partial deformations as well as stress equilibrium equations are formulated and evaluated. Fundamentals for the derivation of such material models for the case of geometrically linear problems can for example be found in Reiner (1968), Visintin (2006), and Altenbach (2012). Applications to the simulation of specific material behaviour are described in Kletschkowski et al. (2001), Bröcker and Matzenmiller (2012, 2013), among others.

In the context of nonlinear continuum mechanics, one famous approach for the derivation of inelastic material models is the multiplicative decomposition of the deformation gradient (see, for instance, Kröner, 1959/1960; Lee, 1969). Here, the elastic parts are constituted by a free energy function and an evaluation of the Clausius–Duhem inequality leads to thermodynamic restrictions imposed on the constitutive equations. Based on these restrictions, the inelastic flow behaviour has to be considered by proper formulations of evolution equations. Different material models including elastic, viscous and plastic properties can be realised with this approach (see, e.g., Dettmer and Reese, 2004; Gurtin and Anand, 2005; Hartmann, 2006; Haupt and Sedlan, 2001; Lion, 2000; Shutov and Kreißig, 2008b; Shutov et al., 2013; Simo and Hughes, 1998).

The idea of relating models of multiplicative inelasticity to rheological elements has been advanced by Lion (1998, 2000) (see also Haupt (2002)). Thereby, the structure of complex material models is explained by rheological elements and connections. Nevertheless, the derivation of constitutive equations is done by the common procedure described above. This approach of illustrating models of multiplicative inelasticity has been adopted by many authors (see, e.g., Ayoub et al., 2010; Dettmer and Reese, 2004; Khajehsaeid et al., 2014; Lejeunes et al., 2011; Pouriayevali et al., 2013; Rodas et al., 2016; Shutov and Kreißig, 2008b).

There are also approaches directly adopting the basic idea of rheological connections. This includes the formulation of kinematic and kinetic connection relations and the evaluation by the help of specific constitutive models for the single elements. In this context, the application of a parallel connection within models at large strains results in a summation of stresses since different elements

* Corresponding author. Tel./fax: +49 371 531 834652.
E-mail address: robert.kiessling@mb.tu-chemnitz.de (R. Kießling).

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Nomenclature

$\mathcal{K}$	Current configuration
$\widetilde{\mathcal{K}}$	Reference configuration
$\hat{\mathcal{K}}, \check{\mathcal{K}}$	Intermediate configurations
$\underline{\underline{C}}$	Right Cauchy–Green tensor (see Eq. (2))
$\underline{\underline{D}}$	Strain rate tensor (see Eq. (4))
$\underline{\underline{F}}$	Deformation gradient
$\underline{\underline{I}}$	Identity tensor
$\underline{\underline{L}}$	Velocity gradient (see Eq. (3))
$\underline{\underline{b}}$	Left Cauchy–Green tensor (see Eq. (2))
$\underline{\underline{\sigma}}$	Cauchy stress tensor
$\underline{\underline{\tau}}$	Kirchhoff stress tensor (see Eq. (11))
$\underline{\underline{\widetilde{T}}}$	Second Piola–Kirchhoff stress tensor (see Eq. (11))
$\widetilde{X}, \hat{X}, \check{X}$	Quantities operating on $\widetilde{\mathcal{K}}$, $\hat{\mathcal{K}}$ and $\check{\mathcal{K}}$, respectively
$\overset{i}{X}$	Quantity of the ith partial deformation
$\overset{S}{\underline{\underline{X}}}$	Symmetric part of a second-rank tensor
$\underline{\underline{X}}'$	Deviatoric part of a second-rank tensor
$\overset{h}{\underline{\underline{X}}}$	Hydrostatic part of a second-rank tensor
$\overline{\underline{\underline{X}}}$	Unimodular part of a second-rank tensor
$\overset{V}{\underline{\underline{X}}}$	Volumetric part of a second-rank tensor
$\overset{G}{\underline{\underline{X}}}$	Distortional part of a second-rank tensor
$\overset{\triangle}{\underline{\underline{X}}}$	Lagrangian time derivative
$\underline{\underline{X}}^{\square}$, $\underline{\underline{X}}^{\hat{\square}}$,	Mixed contravariant-variant Oldroyd rate with respect to $\mathcal{K}$ and $\hat{\mathcal{K}}$, respectively
$f\left(\left(\underline{\underline{X}}\right)\right)$	Tensor valued function of a tensorial argument
$S((\ldots))$	Translation operator (see Eq. (20))
$\langle\ldots\rangle$	Macaulay brackets
$[\ldots]_{(6)}$	Components of a second-rank symmetric tensor represented in a six-row vector (see Eq. (103))
$\{I_k\}_{k=1,2,3}$	System of invariants of a second-rank tensor (see Eq. (1))
J_1, J_2	First and second principal invariant of $\overset{G}{\underline{\underline{C}}}$ (see Eq. (51))
J_3	Third principal invariant of $\underline{\underline{F}}$ (see Eq. (12))
P_{Sp}	Stress power (see Eq. (13))
$\mathrm{d}V$	Differential volume
Φ	Yield function (see Eq. (62))
ρ	Mass density
σ_F	Initial yield stress
φ	Inelastic arc length (see Eq. (65))

are defined with respect to one single partial deformation. This straight forward utilisation is applied for example by Bergström and Boyce (1998, 2000), Boyce et al. (2000), Silberstein and Boyce (2010), Liu and Hoo Fatt (2011), Shim and Mohr (2011), Ge et al. (2014), Abdul-Hameed et al. (2014)). Moreover, the parallel network model proposed by Bergström and Boyce (1998) is already available in the commercial finite element code Abaqus. Thus, a wide range of material models based on a parallel connection of an arbitrary number of rheological elements can be regarded (see Abaqus, 2013; Hurtado et al., 2013). However, this model and its implementation are restricted to a special class of rheological structures, i.e. the parallel connection of different elements.

In contrast, the treatment of the multiplicative decomposition of the deformation gradient, being interpreted as a series connection, is a more challenging task. Here, appropriate stress measures on the connection configuration have to be identified. An early work dealing with this topic is given by Krawietz (1986). Therein, specific connection relations for the parallel and series connections within the framework of multiplicative decomposition of the deformation gradient are formulated. Moreover, Gurtin (2000) and Gurtin and Anand (2005) utilise the principle of virtual power to develop a continuum theory for the elastic-viscoplastic deformation of amorphous solids. Assuming incompressible, irrotational plastic flow, the evaluation of the according macroscopic and microscopic force balances yields relations between the internal forces associated with the responses of the material, which can be interpreted as kinetic relations of rheological connections (see also Gurtin and Anand, 2005; Henann and Anand, 2009). Examples of application can be found in Anand et al. (2009), Ames et al. (2009), and Balieu and Kringos (2015). A general approach for the derivation of models of multiplicative inelasticity by rheological connections is given by Ihlemann (2014). Based on the multiplicative decomposition of the deformation gradient and the stress equilibria, general relations for the stresses are derived. Moreover, specific models for elastic, viscous and plastic material behaviour are formulated. An evaluation of these basic models according to the connection relations yields different elementary models of multiplicative viscoelasticity and elastoplasticity. Bröcker and Matzenmiller (2014) present a similar approach. However, contrary to Ihlemann (2014), they postulate the balance of the stresses and consider the decomposition of the stress power as a result (see also Bröcker, 2014).

The connection relations formulated by Ihlemann (2014) enable the development of an alternative numerical solution procedure. In Landgraf and Ihlemann (2012), the application of Ihlemann's concept to a famous model of multiplicative viscoelasticity of Maxwell type is considered. Besides the analytical derivation of constitutive equations, a numerical implementation directly acting on the connection relations is presented. Instead of solving evolution equations, the connection relations are numerically evaluated by the Newton–Raphson method.

The article at hand addresses the fundamentals of Ihlemann's concept and the alternative numerical implementation by Landgraf and Ihlemann (2012). First, the general connection relations of parallel and series connections are given and the thermodynamic consistency is proved. Next, these relations are specified for the application within nearly-incompressible formulations. Subsequently, individual material models, i.e. the rheological elements, are defined in form of stress–strain relations or stress–strain rate relations.

The approach is illustrated by reviewing an application example. Here, the viscoplastic material model of overstress type with nonlinear kinematic hardening by Shutov and Kreißig (2008b) is examined. The analytical compliance of the constitutive equations, derived by the classical approach on the one hand and by the concept of rheological connections on the other hand, is shown. Furthermore, the numerical implementation based on rheological connections by Landgraf and Ihlemann (2012) is extended. Finally, this procedure is applied to Shutov and Kreißig's material model. The simulation results are compared with the predictions obtained by the standard solution method.

Throughout this article, underscored symbols denotes tensors in $\mathbb{R}^3$, whereby the number of underscores represents the rank of the tensor. Let $\underline{\underline{I}}$ be the second-rank identity tensor. A full system of invariants of a second-rank tensor is defined by

$$I_1\left(\underline{\underline{A}}\right) := \underline{\underline{A}} \cdot\cdot \underline{\underline{I}}, \quad I_2\left(\underline{\underline{A}}\right) := \frac{1}{2}\left[I_1\left(\underline{\underline{A}}\right)^2 - I_1\left(\underline{\underline{A}}^2\right)\right] \quad \text{and}$$
$$I_3\left(\underline{\underline{A}}\right) := I_1\left(\underline{\underline{A}}\right) I_2\left(\underline{\underline{A}}\right) - \frac{1}{3}\left[I_1\left(\underline{\underline{A}}\right)^3 - I_1\left(\underline{\underline{A}}^3\right)\right]. \tag{1}$$

Here, " $\cdot\cdot$ " denotes the double contraction of two second-rank tensors, according to $\underline{\underline{A}} \cdot\cdot \underline{\underline{B}} := \mathrm{tr}\left(\underline{\underline{A}} \cdot \underline{\underline{B}}\right)$. $\mathrm{tr}(\cdot)$ is the trace of a