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A second-order flexibility-based model for steel frames of tapered members

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ABSTRACT

The paper presents a new computer method for nonlinear inelastic analysis of steel frames consisting with members with non-uniform cross-sections. A novel second-order flexibility-based element has been developed. The behaviour model accounts for material inelasticity due to combined bending and axial force, element geometrical nonlinear effects in conjunction with initial geometric imperfections using only one element per structural member. The proposed element formulation combines the power series approach to obtain the general solution of the second-order bending moments with the Maxwell-Mohr method to compute the force-displacement relationship of the general continuously non-prismatic Timoshenko-Euler beam-column element. The method ensures also that the plastic strength interaction requirements are always satisfied in the plastic hinges developed at the ends of the member or within the member length. The second-order elasto-plastic tangent stiffness matrix and equivalent nodal loads vector of non-uniform 2D steel members with semi-rigid connections is developed and the proposed nonlinear analysis formulation has been implemented in a computer program. In order to verify the efficiency and accuracy of the proposed approach, several benchmark problems have been studied and the results prove the performance of the proposed method.

1. Introduction

Non-prismatic members are commonly used in the steel construction industry because of their structural efficiency. During the last decades several analytical and numerical methods have been developed for evaluation of the nonlinear analysis and stability of uniformly tapered beam–column elements and systems [1–32]. All these methods can be classified in three main categories as follows: (1) standard engineering or simple models, based on suitable adaptation of Euler-Bernoulli or Timoshenko beam theories, non-prismatic members being treated as elements of variable cross-section ignoring in this way the tapering effects [1–26]; (2) enhanced models [27–32] able to consider explicitly the equilibrium on the beam lateral surface with an accurate description of the shear stresses at the top and bottom fibres of the beam and (3) advanced two- and three-dimensional finite element models enhanced with advanced nonlinear constitutive laws [33].

Although FEM based software became very popular and can reflect with a high degree of precision the real behaviour of structures, they are limited in the analysis and design of complex structures, usually such approaches requires very fine-grained modelling, and extensive calibration and mesh generation studies leading to complications in the interpretation of results and implies a high computational effort. Hence, researchers have been concerned in developing new advanced analysis methods for facilitating the evaluation of response of structures in practice by using *line element* approaches.

Despite of recognizing the limitations of the simple models in considering some sensitive aspects in the modelling of such particular elements, mainly due to overlooking of the equilibrium on the beam lateral surface (i.e. tapering effect), the simple models are still attractive for practical applications and still represents the focus of intense research efforts [18–26].

For instance for global buckling of tapered frame elements and their effective length factor, analytical solutions based on particular solutions of differential equilibrium equation of Euler-Bernoulli uniform tapered beam-column have been proposed (e.g., [2–4]). These methods solve either the second or fourth order degree of the differential equation considering the displacement as the main unknown and taking into account different particular shapes of variation of cross-section along the member length. More general solutions based on power series or Chebyshev polynomial approaches to obtain elemental stiffness matrix for tapered beams have been

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developed in (e.g., [5–7]). Lateral-torsional buckling analyses of tapered members and systems have been proposed also in several papers (e.g., [8–10]) among others. Moreover, the analytical and finite element formulation can be cast within the framework of displacement based models (e.g., [11–17]) and flexibility-based model (e.g., [18]). A useful brief review about this issue has been given in [18]. It is worth noting that just a few papers have been paid attention to coupling effects of constant axial force (nonlinear geometrical effects) and shear deformation according to Timoshenko's theory on nonlinear behaviour of tapered beam-column elements (e.g., [7]) or to tapered elements with flexible connections as would occur in semirigid frames (e.g., [4,18]).

Tapered beam-column elements studied by many researchers usually is an idealized model. In reality, regardless of the quality of the manufacturing process, steel profiles develop imperfections which could affect the stability of the structure, may cause premature collapse and in general change the nonlinear behaviour both at the element and global level. In order to obtain a structural response closer to the reality, the effect of initial geometrical imperfections on the behaviour of tapered beams and columns has been studied by several researchers (e.g., [2,19]).

Concerning the effects of material nonlinearity several approaches have been developed. Leu et al. [16] proposed a quasi-plastic hinge approach for beam elements with uniform and nonuniform cross-sections through the use of different moment-curvature relations but the element second–order and shear deformation effects are ignored in their analysis. Li et al. [17] proposed a concentrated plasticity model for second-order inelastic analysis of steel frames of tapered members but plastic hinges are allowed to be developed only at the ends of the beam-column element.

Although several advanced models for the nonlinear analysis of tapered steel frames have been developed, some of them mentioned above, several important features, for practical applications, such as the combined effects of element geometrical nonlinearity, initial geometric imperfections, lateral loads applied on member length, shear deformation, semi-rigid behaviour of the connections and the ability to capture plastic hinge development within the element length by using only one line element per structural member are not completely and efficiently developed in the computational methods addressed in the literature.

Within the framework of the standard engineering models, mentioned above, the present work attempts to develop accurate yet computational efficient tools for the nonlinear inelastic analysis of steel frames with tapered members and semi-rigid connections. Essentially, the nonlinear inelastic analysis employed herein takes the advantage of using only one beam-column element per structural member simultaneously considering the effects of second-order geometrical nonlinearities, shear deformation and initial geometric imperfections featuring in this way the ability to be used for practical applications by combining modelling and computational efficiency in conjunction with a reasonable accuracy. Distributed lateral loads acting along the member length can be directly handled into the model without the need to generate additional elements along the member length such that the same topological model could be used both in the linear and the nonlinear analyses. The method ensures also that the plastic strength requirements are always satisfied in the plastic hinges developed within the member length without the need to divide the member into two elements and applying the plastic flow rules at the element ends as in [34] or to make additional operations of static condensation as in [35]. By contrast with the aforementioned approaches, the present element formulation combines the power series approach with the Maxwell-Mohr method to compute the force-displacement relationship at the element level highlighting in this way the flexibility matrix and equivalent nodal loads of the tapered element. The second order differential equilibrium equation is expressed at the element level represented in the natural coordinates, with the rigid body modes removed, and considering as a primary unknown the bending moment the equation is successfully solved by applying the power series approach. The proposed governing equation and the resulted element stiffness matrix and equivalent nodal load vector are of general forms, they can be applied to any variation of sectional shape along member length subjected to uniform axial compressive or tensile forces. Such a formulation allow us to take into account in a more efficient manner the initial geometrical imperfections and member lateral loads, the effects of shear deformation are integrated directly in force-displacement relationships by means of applying the Maxwell-Mohr rule to compute the generalized displacement in the second-order geometrically nonlinear analysis. In this respect the element force fields are described by the second-order bending moments and shear forces derived by solving the second-order differential equation with variable coefficients in the presence of the axial force, member lateral loads and the second-order effects associated with the initial geometric imperfections. In this way the elements of the stiffness matrix and equivalent nodal loads can be obtained analytically and readily evaluated by computing the *correction coefficients* that affect the first-order elastic flexibility coefficients and equivalent nodal loads of prismatic beam-column element. The effect of the transverse shear deformation can be readily included in the element formulation, both in stiffness matrix and equivalent nodal loads. A plastic hinge method is adopted to simulate member plasticity; plastic hinges are assumed to be lumped either at the ends of the beam-column element or along the element length. In the present paper the concentrated plasticity model proposed in [36] for the derivation of force-displacement relation is extended for curved convex yield surfaces and for non-prismatic elements in conjunction with consideration of second order effects, lateral loads and initial geometric imperfections in the bending moment expression. The effect of semi-rigid connections could be included in the second-order plastic hinge analysis using the zero length rotational spring element approach. The efficiency, accuracy and robustness of the analytic procedure and the computer program developed here has been evaluated using several benchmark problems analysed previously by other researchers using independent analytical, numerical and finite element solutions.

2. Formulation of the proposed analysis method

The present formulation is based on the following assumptions: (1) plane section remain plane after flexural deformation; (2) local and torsional buckling do not occur; lateral torsional buckling is prevented; warping and cross-section distortion are not considered and shear distortion is neglected; (3) transverse shear deformations associated to the transverse shear forces are neglected in the plastic constitutive relationships; (4) the element is considered to be continuously non-prismatic with doubly symmetric cross-section; (5) small strain but arbitrarily large displacements and rotations are considered.

As a consequence of the assumption (4) the locus of cross-section centroids coincide with the beam axis avoiding in this way the strong coupling between the bending moment, shear and axial forces allowing in this way to treat the continuously non-prismatic straight beams as beams of variable cross-section with straight line element axis. For those beams with non-continuously variable cross-sections along the member length the beams are divided in a series of continuously non-prismatic elements and treat those segments as continuously non-prismatic straight elements. Nonlinear response of tapered frame structures is mainly dominated by the bending moments and axial loads while shear deformations are relatively small under certain conditions [19,20]. However in the present study the effect of transverse shear deformations over element stiffness are modelled by means of Timoshenko model and some theoretical considerations to evaluate the tapering effect [27] when shear deformations are taken into account are presented. Preventions of induced warping by torsion, cross-section distortion