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Stability and stiffness analysis of a steel frame with an oblique beam using method of least work

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ABSTRACT

For the first time, this paper investigates the stability and stiffness analysis of a single column in a specific case with the connected oblique beam. In this case, the modification factors are analytically derived such that the stiffness of oblique beam is included in the calculation of the well- known parameters G_T and G_B. The effective length factor for such column can be obtained by these modified G_T and G_B. It is noted that the effective length factor of the column in the mentioned specific case is assessed for the first time in this research. In the following, a single span- one story steel frame is investigated to determine the lateral stiffness of the frame. In this frame, the applied lateral load and the structural frame are not on the same plane. Accordingly, it is also focused on the investigation of the effects of hinge existed in the beam as well as the changing of column base connection from fixed to hinged forms. The structure is considered as a 3- dimensional steel frame for the analysis upon the least work principle. All effective factors are taken into account including axial and shear loads as well as bending and torsional moments. At the end of the analysis, a relation is obtained through response surface method. The lateral stiffness can be calculated by the derived relation based on the specifications of the steel frame such as geometrical properties of the employed sections, specification of the used material and deviation angle of the beam.

1. Introduction

Stability analysis is one of the most important kinds of analyses which should be considered by engineers for analyzing the structures [1]. The elements of steel frames extensively used in buildings due to their structural efficiency have often small sectional areas because of the high strength of steel. This fact results in the higher probability of the global and local bucklings of elements in steel frames. Therefore, stability analysis is one of the primary concerns in design process of steel structures and developing a practical, effective and reliable approach to examine the stability of steel frames is a crucial challenge for engineers.

The effective length method has been ordinarily used by engineers for over 50 years in stability analysis of the columns in steel structures. The deficiencies of this method has resulted in presenting a new approach, so called “direct analysis method” in the last version of American Institute of Steel Constructions (ASIC) [2]. A problem with the effective length method is that if the beam connected to the column is oblique, then no solution is found in the literature for obtaining the effective length factor of the column. Although few studies on columns with elastic and rigid oblique restraint have been done, but all of these studies have been done with numerical methods like finite element method; therefore, they can only be considered for certain and not any cases. Of the mentioned researches, it can be referred to Trahair and Rasmussen researches [3,4,5,6]. In the first section of this research, it is attempted to calculate effective length factor based on the mathematical methods for the cases of oblique beam connected to the column. Providing appropriate lateral stiffness for the structure has significant effects on its seismic performance. This stiffness can be easily calculated if the lateral load and frame are all in the same plane. However, it is not straightforward while the beams connected to the columns are oblique.

In the second section of this research, lateral stiffness is calculated for a one-span one story steel frame considering that the beam connecting two columns is oblique, and the lateral load and frame are not on a plane.

In this section the important question is that while the desired responses can be obtained easily through finite element softwares, why the structures should be assessed by this method? The answer is that despite the heaviness of presented formulas, they have undeniable advantages in comparison to the numerical methods. That is in the latter, the response of each problem is obtained as per specified input values; therefore, the behavior trend of the structure cannot be investigated

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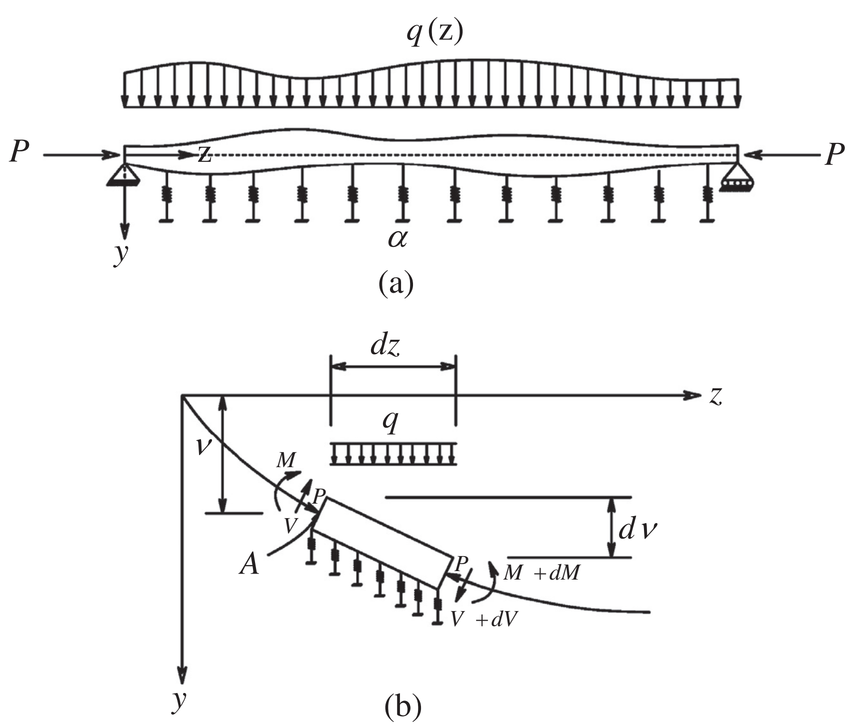

Fig. 1. The parameters used in the differential equation of buckling

well by these methods. The presented method offers a proper understanding of the behavior of the structures with rigid connection of oblique beam to column, even though with a one-story one- span structure. This understanding is much more valuable than the response of a particular problem. Moreover, the relations presented in the final section of this article have been obtained by the aid of these complicated formulas. While intensive calculation effort is needed each time in solving the problem by numerical methods, it is enough only once in the suggested method.

2. Solving the differential equation of buckling for a column

Consider a structural element subjected to an axial load P and a distributed load, q on the elastic bed as shown in Fig. 1(a). The governing differential equation of buckling for this element is generally expressed as follows:

$$EI_x\nu^{iv} + P\nu'' + \alpha\nu = q \quad (1)$$

where E is elastic modulus; I_x is moment of inertia; alpha is bed's elastic modulus and v is lateral displacement. This equation considers all factors which are effective on the stability of the element. Its parameters are presented in Fig. 1.

However, the columns, which are ordinarily used in the structures, are not located on the elastic beds. Moreover, no distributed load is applied along the column. Therefore, Eq. (1) is re-written for such columns as follows:

$$EI_x\nu^{iv} + P\nu'' = 0 \quad (2)$$

Solving the above differential equation results in the following solution:

$$\nu = A + Bz + C\sin kz + D\cos kz, \quad k^2 = \frac{P}{EI} \quad (3)$$

Obviously, four boundary conditions are needed for obtaining accurate values of ν. In this article, the buckling of columns is investigated in the steel frame and therefore no ideal assumption (such as hinged, roller or fixed end) should be considered for their boundaries. Fig. 2 is used for better understanding of the investigated column.

Fig. 3 is applied to express the boundary conditions in the problem.

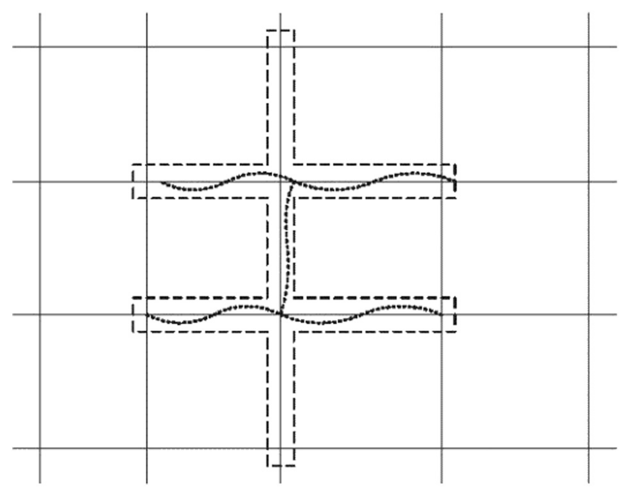

Fig. 2. Configuration of the element used in the steel frame for buckling analysis.

In this figure, both ends of the studied element can have either torsion or transmission movement. It should be noted that the springs used in each ends indicate the stiffness of the front and back beams at the end of column. In the other words, each spring contain the stiffness of two beams.

Boundary conditions of the considered problem are presented as follows:

$$\text{at } z = L \rightarrow \begin{array}{l} -EI\upsilon''' - P\upsilon' = \beta_B \upsilon \\ -EI\upsilon'' = -\alpha_B \upsilon' \end{array} \quad \text{at } z = 0 \rightarrow \begin{array}{l} -EI\upsilon''' - P\upsilon' = \beta_T \upsilon \\ -EI\upsilon'' = -\alpha_T \upsilon' \end{array}$$

The first line shows the equalization of the bending moment of column end with the moment created in the torsional spring. The second line shows the equalization of the shear force of column end with the force created in the transitive spring. The parameters used the mentioned boundary conditions are as follows: