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Reduced stiffness of composite beams considering slip and shear deformation of steel

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ABSTRACT

In this paper, a theory is developed for analysis of steel-concrete composite beams with interfacial slip and shear deformation in steel considered. Closed-form solutions are derived for simply supported composite beam under uniform and mid-span concentrated load respectively. Examples are given to compute various quantities of beams. Comparing the results calculated by closed-form solutions with those by ANSYS, good agreements are found. Finally, an explicit formula for the equivalent bending stiffness of the composite beams is found by which the deflections of the composite beams can be calculated as if they were common Bernoulli beams.

1. Introduction

Slip occurs in the concrete slab-steel beam interface under loads. Newmark [1] was the first to propose an elastic analysis theory considering slip, his theory was used to calculate the deflection and analyze the stresses of composite beams. R.P. Johnson [2] proposed a simplified formula to calculate the deflection. He suggested that the deflection of composite beams with partial shear connection can be interpolated between the deflections of pure steel beams and of the composite beams with complete shear connection. Jianguo Nie [3] presented a reduced stiffness method to calculate the deflection of steel-concrete composite beams. Genshu Tong [4] introduced a composite factor to compute the reduced stiffness due to the interfacial slip.

The steel section in a steel-concrete composite beam is small compared with a pure steel beam. And in a composite beam, only the web of the steel beam is counted on to resist shear. Thus the shear stress and the deformation in the web of I-steel section are much larger than those in pure steel beams under the same loads, which could have a non-ignorable effect on the deflection. Gianluca Ranzi [5] proposed a finite analysis model for composite beams considering partial shear deformation, coupling an Euler-Bernoulli beam for the reinforced concrete slab to a Timoshenko beam for the steel beam. Feifei Sun [6] introduced a closed-form solution for steel-concrete composite beams with slip, shear lag and shear deformation. However, due to the large number of undetermined coefficients, no one proposes a simplified formula to calculate the deflection of steel-concrete composite beams with both interfacial slip and shear deformation in steel. In this paper, the Timoshenko beam theory is introduced to the traditional composite beam theory in Ref. [4] to consider shear deformation as well as slip. And equilibrium differential equations are derived and solved in closed form. Finally an explicit equation for the reduced stiffness of the composite beam is obtained which includes the effect of interfacial slip and steel shear deformation. The obtained equation can be used to compute the deflection of the composite beam as if it were a common Bernoulli beam.

2. Basic assumptions and notations

The following assumptions are adopted:

1) The composite beam is elastic.
2) The relation of the interfacial shear to interfacial slip is linear.
3) The concrete slab and the steel beam have the same deflection.
4) The Euler-Bernoulli beam theory is used for the concrete slab and the Timoshenko beam theory is for the steel beam.

Assuming that the concrete slab and the steel beam are represented by 2 beam axes. The axis x is in the longitudinal direction, one supported end is set as the origin of the coordinates and the plane yz is the cross section plane as shown in Fig. 1. The internal force directions are shown in Fig. 2. u, w, v are the displacements in directions of x, y, z-axes respectively. The axial force is positive in tension. And the bending moment is positive if the y-positive side is in tension. Meanwhile, the shear force is positive if it makes the micro-body rotate clockwise. And q is positive if the load points to the positive direction of y axis.

With the Timoshenko beam theory used to consider the shear deformation, the deflection of the steel beam is composed of two parts,

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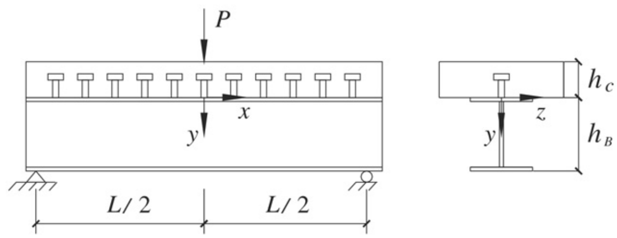

Fig. 1. Simply-supported composite beam model under concentrated load.

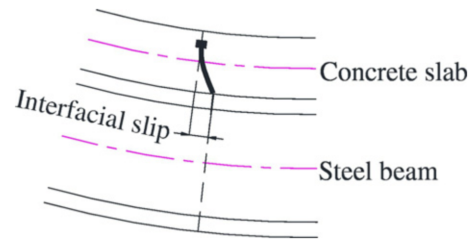

Fig. 3. The deformation of the stud after interfacial slip.

i.e., $w = w_b + w_s$, where $O_6{}' = \frac{h_{sc}B_1 - B_3\beta^2}{\beta^2 - B_1}\beta \sinh(\frac{\beta l}{2})$ represents the bending deflection and $E_s = 206\text{kN/mm}^2, E_c = 30\text{kN/mm}^2, G_s = 79\text{kN/mm}^2$ the shear deflection. The shear deformation doesn't account for the longitudinal displacement, so it doesn't increase the slip deformation.

The horizontal displacement of the upper surface of the steel beam is represented by $k = 500\text{N/mm}^2$ as:

$$u_1 = u_{10} + h_{s1} w_b' \tag{1a}$$

where u_{10} is the horizontal displacement of the steel beam centroid, and h_{s1} is the distance between the steel centroid and the steel-concrete interface (upper surface of the steel).

The horizontal displacement of the lower surface of the concrete slab is represented by u_2 as:

$$u_2 = u_{20} - h_{c2} w' = u_{20} - h_{c2}(w_b' + w_s') \tag{1b}$$

where u_{20} is the horizontal displacement of the concrete slab centroid, and h_{c2} is the distance between the slab centroid and the steel-concrete interface (lower surface of the concrete).

The slip s in the steel-concrete interface is:

$$\begin{aligned} s &= u_1 - u_2 = u_{10} - u_{20} + h_{s1} w_b' + h_{c2}(w_b' + w_s') \\ &= s_0 + h_{sc} w_b' + h_{c2} w_s' \end{aligned} \tag{2}$$

where $h_{sc} = h_{s1} + h_{c2}$, and $s_0 = u_{10} - u_{20}$ is the relative slip displacement between the concrete centroidal axis and the steel one.

The interfacial shear force per unit length in the studs q_u is given by: (The positive direction is shown as Fig. 3.)

$$q_u = ks = k(s_0 + h_{sc} w_b' + h_{c2} w_s') \tag{3}$$

where k is the slip stiffness in the interface between the steel beam and the concrete slab.

3. Theoretical development

N_s, M_s are the axial force and the bending moment of the steel beam respectively. N_c, M_c are the axial force and the bending moment of the concrete slab respectively. The concrete slab is an Euler-Bernoulli beam so that its shear deformation is neglected. Therefore M_s is obtained by w_b and M_c is obtained by w.

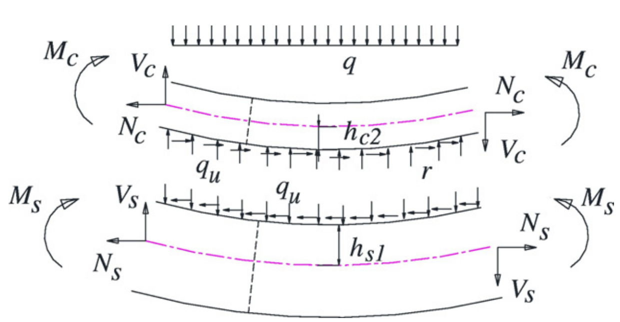

Fig. 2. The deformation and forces of the composite beam.

N_s, M_s, N_c, M_c are expressed as:

$$M_s = -E_s I_s w_b'', \quad N_s = E_s A_s u_{10}' \tag{4a}$$

$$M_c = -E_c I_c w'', \quad N_c = E_c A_c u_{20}' \tag{4b}$$

where E_sI_s and E_sA_s are the bending stiffness and axial compression stiffness of the steel beam around the centroid axis, and E_cI_c and E_cA_c are the bending stiffness and axial compression stiffness of the concrete slab around the centroid axis.

V_s is the shear force in steel section, and V_c is the shear force in concrete section. They can be obtained from the equilibrium of the infinitesimal elements of the steel beam and the concrete slab as:

$$\frac{dM_s}{dx} = V_s - h_{s1} q_u = -E_s I_s w_b''', \tag{5a}$$

$$\frac{dV_s}{dx} = -r, \tag{5b}$$

$$\frac{dM_c}{dx} = V_c - h_{c2} q_u = -E_c I_c w''', \tag{5c}$$

$$\frac{dV_c}{dx} = -q + r, \tag{5d}$$

$$\frac{dN_s}{dx} = q_u = E_s A_s u_{10}'', \tag{6a}$$

$$\frac{dN_c}{dx} = -q_u = E_c A_c u_{20}'' \tag{6b}$$

where r is the compression interactive force between the concrete slab and the steel section. Adding Eqs. (5b) and (5d)

$$\frac{dV}{dx} = \frac{d}{dx}(V_s + V_c) = -q \tag{7}$$

Adding Eqs. (5a) and (5c)

$$-E_s I_0 w_b''' - E_c I_c w_s''' = V - q_u h_{sc} \tag{8a}$$

where $I_0 = I_s + I_c/\alpha_E, \alpha_E = E_s/E_c$. Multiplying Eq. (6a) by E_cA_c and Eq. (6b) by E_sA_s, then subtracting one by the other, one obtains:

$$E_s A_s E_c A_c (u_{10}'' - u_{20}'') = q_u (E_s A_s + E_c A_c) \tag{8b}$$

Derivation of Eq. (8a) once and rearranging Eq. (8b), the following equilibrium differential equations are obtained:

$$E_s I_0 w_b^{(4)} + E_c I_c w_s^{(4)} - q_u' h_{sc} = q \tag{9a}$$

$$E_s A_0 s_0'' = q_u \tag{9b}$$

where $A_0 = \frac{A_s A_c}{\alpha_E A_s + A_c}$.