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# Axially restrained steel beams with web openings at elevated temperatures, part 2: Development of an analytical method

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ABSTRACT

The companion paper has presented the results of an extensive numerical study to investigate the behaviour of axially restrained steel beams with web opening, and to identify how the key quantities of the behaviour are controlled by different design parameters, including size and location of the opening, level of axial restraint, load ratio and temperature distribution. This paper uses the numerical simulation results to derive an analytical method to evaluate the key quantities, including the initial slope of the beam axial force – temperature relationship (rate of increase in compressive force with respect to temperature increase), point of initial buckling of the Tee-section under compression, transition temperature from compression to tension, the maximum tensile force and the corresponding temperature.

For practical design, the beam's limiting temperature is calculated without consideration of axial restraint. At this temperature, since there may be considerable axial tensile force in the beam with web opening, it is important to check the capacity of the connections against the maximum tensile force, which can be calculated accurately using the proposed method.

## 1. Introduction

The companion paper [1] has presented the results of an extensive numerical simulation on steel beams with web openings in order to investigate the effects of openings on axially restrained steel beams at elevated temperatures. The examined parameters included opening shape (circle, square, rectangle and elongated circle), opening size (height 0.50 $h$, 0.67 $h$ and 0.75 $h$, length $0.5d_o$, $1d_o$, $1.5d_o$, $2d_o$, $2.5d_o$ and $3d_o$) and opening position (at 1/6th beam span, 1/3rd beam span or mid-span), load ratio (0.3, 0.5 and 0.7), level of axial restraint (0 to infinity) and cross-section temperature distribution profile. The steel section was UB457 × 152 × 60. The steel grade was S275 with a Young's modulus of 210,000 N/mm$^2$. Note that even though the steel section was kept the same for all simulations, changes in the opening shape and size allowed for different tee-section properties to be simulated.

The simulation results of the parametric study have shown that an axially restrained steel beam with web opening experiences the following stages of behaviour: initial development of compression force due to restrained thermal expansion, followed by initial buckling/fielding failure at the opening under combined axial compression, bending moment and shear after which compression unloading occurs. At the beam transition temperature, the axial force in the beam changes from compression to tension. During the catenary action stage, the magnitude of the maximum axial tensile force increases as the level of axial restraint increases. The final stage of the beam behaviour is controlled by the plastic axial capacity of the beam at high temperatures.

In many cases, axially restrained steel beams with web opening behave as solid beam and the current steel beam design method, based on axially unrestrained steel beam, can be used without further complication provided the beam's temperature is lower than the limiting temperature of the unrestrained beam.

However, if the opening is large and the tee-section is slender and it is placed at high bending moment region, the beam transition temperature (at which the beam returns to pure bending) may be much lower than the failure temperature calculated using the current design method which gives the limiting temperature of the identical unrestrained beam. It would not be practical to take this transition temperature as the beam's limiting temperature. Fortunately, for axially restrained steel beams with web opening, the beam may be able to survive much high temperatures the than transition temperature. Therefore, it is possible for the design limiting temperature of the axially restrained beam to be much higher than the transition temperature. However, the restrained beam will be in catenary action and it is important that the connections and the surrounding structure are able to resist the catenary tension force in the beam at the design limiting temperature.

The simulation results have revealed that when the axial force in the top tee-section becomes zero, the total axial tension force in the beam is at its maximum value. At the same temperature, the tensile force in the bottom tee-section reaches the maximum tensile capacity. Hence the maximum axial tension force in the beam is the same as the tensile

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**Notations**

| | |
|---|---|
| $A$, $A_o$ | Cross-sectional area of I-section and perforated I-section respectively |
| $A_{o,v}$ | Shear area of perforated I-section |
| $A_t$ | Cross-sectional area of tee-section |
| $B_f$ | Width of flange |
| $d$ | Depth between flanges of I-section |
| $d_c$ | Distance between centroids of tee-section |
| $d_o$ | Depth of opening |
| $d_t$ | Depth of tee-section |
| $E$ | Modulus of elasticity |
| $E_\theta$, $E_{e,\theta}$ | Modulus/effective modulus of elasticity at temperature θ |
| $f_{crit,\theta}$ | Critical buckling stress of section with opening at temperature θ |
| $f_{n,\theta}$ | Average direct stress over cross-section at temperature θ |
| $f_{p,\theta}$ | Proportional limit strength at temperature θ |
| $f_{vn,\theta}$ | Reduced shear strength due to axial compressive force at temperature θ |
| $f_y$, $f_{y,\theta}$ | Yield strength, Yield strength at temperature θ |
| $h$ | Overall height of I-section |
| $I_t$ | Second moment of area of tee-section |
| $K_B$ | Axial stiffness of steel beam at ambient temperature |
| $K_{B,\theta}$ | Axial stiffness of beam at temperature θ |
| $K_{o,\theta}$ | Axial stiffness of perforated segment of beam at temperature θ |
| $K_r$ | Axial restraint stiffness |
| $K_{t,\theta}$ | Axial stiffness of tee-section at temperature θ |
| $K_\theta$ | Axial stiffness of member/segment at temperature θ |
| $k_{y,\theta}$ | Reduction factor for yield strength at temperature θ |
| $l$ | Span of beam |
| $l_o$ | Length of opening |
| $l_{t,cr}$ | Critical length of tee-section |
| $M_{o,b,\theta}$ | Buckling moment capacity of section with opening at temperature θ |
| $M_{o,b,N,\theta}$ | Buckling moment capacity of section with opening reduced to allow for axial compressive force at temperature θ |
| $M_{o,b,\theta}$ | Buckling bending moment capacity of section with opening at temperature $\theta$ |
| $M_{o,sd}$ | Applied bending moment at the centreline of opening |
| $M_{sd}$ | Applied bending moment |
| $M_{t,,pl}$ | Plastic moment capacity of tee-section |
| $N$ | Axial force |
| $N'$ | Initial compressive force in tee-section due to global bending moment |
| $N_{o,b,\theta}$ | Axial compressive resistance of section with opening at temperature θ |
| $N_{o,pl}$ | Plastic axial capacity of perforated I-section |
| $N_{o,pl,\theta}$ | Plastic axial capacity of perforated I-section at temperature θ |
| $N_{o,w,pl,\theta}$ | Plastic axial capacity of perforated web at temperature θ |
| $N_{T,\max}$ | Maximum catenary tensile force |
| $N_{T,\max,partial}$ | Maximum catenary tensile force in beam with partial axial restraint |
| $N_{T,\max,full}$ | Maximum catenary tensile force in beam with full axial restraint |
| $N_{t,b}$ | Axial bucking resistance of tee-section |
| $N_{t,cr,\theta}$ | Critical elastic Euler buckling load of tee-section at temperature θ |
| $N_{t,pl,\theta}$ | Plastic axial capacity of tee-section at temperature θ |
| $N_{t,b,\theta}$ | Axial bucking resistance of tee-section at temperature θ |
| $N_\theta$ | Axial force at temperature θ |
| $P.N.A$ | Plastic neutral axis |
| $RF$ | Reduction factor |
| $t_f$, $t_w$ | Thickness of flange, Thickness of web |
| $V_{o,pl,\theta}$ | Plastic shear capacity of perforated I-section at temperature θ |
| $V_{o,pl,N,\theta}$ | Plastic shear capacity of perforated I-section reduced for axial compressive force at temperature θ |
| $V_{o,sd}$ | Applied shear force at the centreline of opening |
| $W_{o,pl}$ | Plastic section modulus of perforated I-section |
| $x_o$ | Distance of opening centreline from support |
| $\alpha$ | Column imperfection factor or coefficient of thermal expansion |
| $\Delta\theta$ | Temperature difference |
| $\delta_{mec}$ | Mechanical displacement |
| $\delta_{th}$ | Thermal expansion |
| $\delta_v$ | Vertical deflection |
| $\varepsilon_{max}$ | Maximum strain |
| $\varepsilon_{p,\theta}$ | Strain at proportional limit strength at temperature θ |
| $\varepsilon_{th}$ | Thermal strain |
| $\varepsilon_{y,\theta}$ | Yield strain at temperature θ |
| $\theta$ | Temperature |
| $\theta_f$ | Failure temperature of unrestrained beam |
| $\theta_{tr}$ | Transit temperature of axially restrained beam |
| $\overline{\lambda}_{t,\theta}$ | Non-dimensional slenderness of tee-section at temperature θ |
| $\lambda_t$ | Geometrical slenderness ratio of tee-section |
| $\rho_R$ | Axial restraint ratio |
| $\rho_N$ | Tee-section load ratio |
| $\sigma$ | Direct stress |
| $\overline{\nu}$ | Coupled shear capacity ratio |
| $\chi_{t,\theta}$ | Reduction factor for buckling of tee-section at temperature θ |

capacity of the bottom tee-section at the temperature when the compression force in the top tee-section is zero.

Based on the above discussions, this paper will present developments of an analytical method to calculate the following quantities:

1. The transition temperature
2. The catenary tensile force variation at temperatures higher than the limiting temperature of an identical unrestrained beam. This temperature is the limiting temperature in the current design method.

The transition temperature informs the designer whether or not this temperature is acceptable. If this temperature is used as the limiting temperature, it is not necessary to consider axial force in the beam. If this transition temperature is too low and the beam designed for higher limiting temperatures, the catenary tensile force in the beam should be calculated using the results from (2) to check the connections and the surrounding structure.

## 2. General behaviour

Fig. 1 shows the general axial force – temperature relationships of an axially restrained beam with web opening. Note that the limiting temperature of the identical unrestrained beam (vertical line) is higher than the transition temperature of the restrained beam (point B).

The approximate analytical curve is as follows: linear increase in compression force until buckling failure of the top tee-section (line O-A), linear change from the maximum compression (point A) to the maximum tension of the bottom tee-section at point C through the transition temperature (point B). The beam tensile force stays constant (line CD) and intersects the total tensile capacity of the beam at elevated temperatures (at point E). Afterwards, the beam tensile force variation follows the total plastic tensile capacity of the beam with increasing temperature.