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A consistent methodology for the out-of-plane buckling resistance of prismatic steel beam-columns

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ABSTRACT

This paper presents a design proposal for the out-of-plane buckling resistance of prismatic beam-columns subject to axial compression and uniaxial major-axis bending that was developed based on the well-known Ayrton-Perry format. Firstly, the relevant theoretical background is summarized, closely following the theoretical derivation performed by Szalai and Papp (2010). Secondly, the required transformations for the engineering application of the design procedure are detailed and extended to arbitrary bending moment distributions. Appropriate generalized initial imperfection factors for the out-of-plane buckling of beam-columns are defined so as to achieving complete consistency across the stability verifications for columns, beams and beam-columns. The proposed procedure is subsequently validated against a large set of advanced numerical simulations. A good agreement was found between the numerical results and the estimates provided by the proposed design procedure, both in terms of the overall trend and the specific quantitative results. Based on a statistical assessment, the comparison with the interaction expression of Eurocode 3 (2005) (method 2) showed that this proposal slightly outperforms the Eurocode procedure, both in terms of average values and dispersion of results.

1. Introduction

Steel skeletal structures are often designed with individual members subject to major-axis bending and axial force (see Fig. 1). The behaviour of such members results from the combination of both action effects and varies with slenderness. At low slenderness, the load-carrying capacity is governed by cross sectional resistance. With increasing slenderness, the geometrically non-linear effects can no longer be ignored, and out-of-plane (flexural or flexural-torsional) buckling may trigger failure. For intermediate slenderness, instability usually occurs in the inelastic range of the material. In the high slenderness range, instability is essentially an elastic phenomenon.

The interactions between instability and plasticity in beam-columns lead to a very complex 3D behaviour that is not easily amenable to design procedures with a consistent and transparent mechanical basis. Indeed, the resistance of beam-columns is generally checked with interaction formulae that combine the ultimate strengths of the member either as a concentrically loaded column or as a beam under uniaxial bending. Interaction formulae are typically developed either: (i) as modifications to formulae derived from an elastic analysis, with more or less empirical factors whose complexity depends on the desired accuracy and range of validity, or (ii) on a wholly empirical basis [4]. Table 1 shows two representative examples of codified interaction formulae for beam-columns subject to axial compression and major axis bending.

AISC [5] provides an interaction approach for the stability verification of beam-columns with doubly or singly symmetric cross-sections as given in Table 1. The interaction equations represent a lower bound of the resistance [6]. The verification encompasses the beam and column verifications as extreme cases and thus accounting for the limit states of yielding, flexural and/or torsional buckling, flange local buckling, and web local buckling. However, the approach has been reported to be over-conservative for members loaded with major axis bending moment and compression, which are prone to out-of-plane failure [4, 6]. Section H1.3 from AISC [5] gives an alternative equation for the verification of doubly symmetric rolled compact members subject to single axis bending and compression (AISC Commentary [6]).

Focusing on the Eurocode 3 [2] implementation, the interaction factors are established on the basis of the concept of equivalent moment and the amplification of the bending effects as a function of the normalized level of applied axial force, including extensive calibration for proper account of the plasticity effects [7]. However, from the point of view of mechanical consistency and transparency, the resulting interaction formulae are hardly satisfactory, since:

- as a two-step procedure that depends on the buckling resistances of the member in bending only and in compression only, they require successive statistical calibrations: first, an independent calibration

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Notations

Latin upper case letters

A	cross-sectional area
C_1	factor accounting for non-uniform bending moment distributions in the elastic critical moment
C_b	lateral-torsional buckling modification factor
C_{bc}	factor accounting for non-uniform bending moment distributions in the elastic critical moment including compression effect;
E	modulus of elasticity
G	shear modulus
I_t	St. Venant torsional constant
I_y	moment of inertia y-axis
I_w	warping constant
I_z	moment of inertia z-axis
L	length
M_{cr}	elastic critical bending moment
$M_{cr,N}$	elastic critical bendinηg moment including the effect of compression force
$M_{cr,nu}$	elastic critical bending moment for non-uniform bending moment distribution
$M_{cr,N,nu}$	elastic critical bending moment including the effect of compression force for non-uniform bending moment distribution
M_{cx}	factored lateral-torsional buckling strength
$M_{r.}$	maximum bending moment design value
$M_{y,Ed}$	maximum bending moment design value
$M_{y,Rd.}$	major axis bending moment resistance
M_y	major axis bending moment
N	compressive force
$N_{cr,x}$	elastic critical force associated with pure torsional buckling
$N_{cr,z}$	elastic critical force associated with pure flexural buckling about minor axis
N_{Ed}	maximum axial design values acting on the member
$N_{b,Rd.}$	factored buckling strength
P_{co}	factored buckling strength
P_r	maximum axial design values acting on the member
W_y	elastic section modulus relative to y-axis
W_w	warping modulus
W_z	elastic section modulus relative to z-axis

Latin lower case letters

$\bar{e}_0$	equivalent initial geometrical imperfection
k_{yy}	interaction factor
k_{zy}	interaction factor
f_y	yield stress
r_0	polar radius of gyration
$v(x)$	transverse displacement along y axis
$v_0\,(x)$	initial transverse displacement
$\hat{v}_0$	amplitude of initial imperfection
$\hat{v}$	amplitude of transverse displacement along y axis
$w(x)$	transverse displacement along z axis
$w_0\,(x)$	initial transverse displacement

Greek lower case letters

β_N	compression factor
η_{BC}	generalized initial imperfection factor for flexural-torsional buckling of beam-columns
η_{LT}	generalized initial imperfection factor for lateral-torsional buckling
η_z	generalized initial imperfection factor for minor axis flexural buckling
$\theta(x)$	twist rotation
$\hat{\theta}$	amplitude of twist rotation
$\theta_0(x)$	initial twist rotation
$\hat{\theta}_0$	amplitude of initial twist rotation
λ_{BC}	normalized slenderness for beam-columns
λ_{LT}	normalized slenderness for lateral-torsional buckling
λ_z	normalized slenderness for minor axis flexural buckling
φ	over strength factor
χ_{BC}	reduction factor for flexural-torsional buckling of beam-columns
χ_{LT}	lateral-torsional buckling reduction factor
χ_y	major axis buckling reduction factor
χ_z	minor axis buckling reduction factor
ψ	end moment ratio

of the imperfection factors for columns and beams and then a calibration of the interaction factors; and

- for class 1 and class 2 cross-sections (plastic interaction), the proposed expressions for the interaction factors (both for method 1 and for method 2) have no physical meaning.

From a practical point of view, the downside to the wide range of cases covered by the EC3-1-1 [2] interaction expressions resulted in long procedures for the determination of the interaction factors, which are especially burdensome when used for preliminary sizing of the members.

The EC3-1-1 [2] design rules for columns and beams are based on the buckling curve approach. For columns, the design procedure is established on the solution of the differential equation of a pin-ended compressed member with an initial sinusoidal imperfection for the limiting condition of first yield at the critical cross-section (mid-span), cast

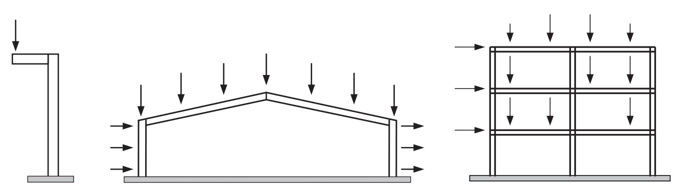

Fig. 1. Steel members subjected to bending and axial force [3].

Table 1
Interaction formulae from representative design codes.

AISC (2010) [5]	$\frac{P_r}{P_c}+\frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right)\leq 1.0$ for $\frac{P_r}{P_c}\geq 0.2$
	$\frac{P_r}{2P_c}+\left(\frac{M_{rx}}{M_{cx}}\right)\leq 1.0$ for $\frac{P_r}{P_c}<0.2$
	$\frac{P_r}{P_{cy}}\left(1.5-0.5\frac{P_r}{P_{cy}}\right)+\left(\frac{M_{rx}}{C_b M_{cx}}\right)^2\leq 1.0^{*}$
Eurocode 3-1-1 [2]	$\frac{N_{Ed}}{\chi_y N_{Rd}}+k_{yy}\frac{M_{y,Ed}}{\chi_{LT}M_{y,Rd}}+\leq 1.0^{**}$
	$\frac{N_{Ed}}{\chi_z N_{Rd}}+k_{zy}\frac{M_{y,Ed}}{\chi_{LT}M_{y,Rd}}+\leq 1.0^{**}$

P_r, N_{Ed} - the maximum axial design values acting on the member.
M_r, $M_{y,Ed}$ - the maximum bending moment design values acting on the member.
P_c, P_{cy} - factored buckling strengths in compression.
N_{Rd} – compression resistance.
M_{cx} - factored lateral-torsional buckling strength.
$M_{y,Rd.}$ – major-axis bending moment resistance.
χ_y, χ_z – flexural buckling reduction factors for major and minor axis.
χ_{LT} – lateral-torsional buckling reduction factor.
k_{yy} k_{zy} are interaction factors.
C_b – lateral-torsional buckling modification factor.
* Alternative verification for doubly symmetric rolled compact members subject to single axis flexure and compression.
** The terms required only to account for the shift of the centroidal axis in class 4 cross-sections have been omitted.