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Prediction of elastic and acoustic behaviors of calcarenite used for construction of historical monuments of Rabat, Morocco

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ABSTRACT

Natural materials (e.g. rocks and soils) are porous media, whose microstructures present a wide diversity. They generally consist of a heterogeneous solid phase and a porous phase which may be fully or partially saturated with one or more fluids. The prediction of elastic and acoustic properties of porous materials is very important in many fields, such as physics of rocks, reservoir geophysics, civil engineering, construction field and study of the behavior of historical monuments. The aim of this work is to predict the elastic and acoustic behaviors of isotropic porous materials of a solid matrix containing dry, saturated and partially saturated spherical pores. For this, a homogenization technique based on the Mori–Tanaka model is presented to connect the elastic and acoustic properties to porosity and degree of water saturation. Non-destructive ultrasonic technique is used to determine the elastic properties from measurements of P-wave velocities. The results obtained show the influence of porosity and degree of water saturation on the effective properties. The various predictions of Mori–Tanaka model are then compared with experimental results for the elastic and acoustic properties of calcarenite.

1. Introduction

The study of the role of porosity, geometry of the pore space and fluid flow in the elastic and acoustic behaviors of porous media is fundamental to understanding phenomena in different fields (e.g. geophysical subsurface and reservoirs, seismology, engineering, monuments, and construction). Indeed, presence of porosity and fluid results in a modification to elastic and acoustic properties which can be modeled by homogenization techniques.

The state of saturation of the natural rock influences the evolution of its elastic and acoustic properties and has been the subject of numerous works, many of which are related to prediction of elastic and acoustic properties of a multiphase porous rock and generally deal with a fully saturated state in comparison to a dry state. The propagation of an elastic wave in a biphasic medium (solid–liquid) was described by Biot (1956), who introduced inertial coupling concepts to the fluid-solid and relative movements of these two phases.

The relationship between the partial saturation and the acoustic properties of rock is more complex. Its analysis requires following experiment very rigorously and involves, from a theoretical point of view, the mechanisms at pore scale and fluid distribution concepts in porous networks. The acoustics of porous and heterogeneous media aims to characterize the seismic waves by synthesizing between the rigor of the laws of mechanics and the natural disorder of porous media (Pham, 2003; Wang and Li, 2007).

Many fields of geophysical and geological investigations are almost invariably faced with the need to determine the properties such as porosity, degree of cracking or fracturing, degree of water saturation, permeability, thermal conductivity, temperature and pressure of fluid in different parts of the crust. Studies have focused on the influence of rock properties and fluids on geophysical, petrophysical and thermal measurements (Gregory, 1976; Han et al., 1986; Klimentos, 1991; Popov et al., 2003; Samaouali et al., 2010; Rahmouni et al., 2013, 2014a, b).

The first models devoted to calculating the effective elastic properties of composite materials were proposed by Voigt (1889) and Reuss (1929). The Voigt approximation provides an upper bound, while the Reuss approximation is a lower bound for the effective moduli. Hashin and Shtrickman (1963) developed a

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variational approach that can identify, for isotropic media, a narrower range for the effective moduli than the average of Voigt (1889) and Reuss (1929) approximations. These models lead to a broad framework of elastic moduli of composite media, mainly when phases presented have very different elastic properties. This remark is particularly appropriate that we seek, in the present case, to determine the effective elastic properties of a rock whose pore space is either empty or unsaturated or saturated with fluid.

The models such as Eshelby, dilute, Mori–Tanaka, self-consistent, Ponte Castaneda and Willis are characterized by taking into account the geometry of the constituent phases of porous media. Some authors have applied these models to describing the behavior of porous media. For example, Guéguen et al. (1997) used the differential self-consistent model to determine the elastic moduli of porous media. Xu (2004) applied changing scale methods to modeling the behavior of unsaturated porous media whose solid phase consists of a linearly elastic material. The prediction of the hydromechanical behavior of clays compacted by homogenization models was studied by Barboura (2007). Sarout (2006) applied the Mori–Tanaka model to describing the propagation of elastic waves in dry argillaceous rocks. Miled et al. (2011) studied the prediction of elastic moduli of isotropic porous materials constituting a solid matrix based on well-known mean-field Eshelby-based homogenization schemes.

Many studies have been devoted to the modeling of effective elastic and acoustic properties of partially saturated porous media (Gregory, 1976; Mavko and Hoeksema, 1994; Cadoret, 1995; Le Ravalec et al., 1996; Dvorkin et al., 1999; King et al., 2000; Li et al., 2001; Toms et al., 2006). Gassmann (1951)'s equations provide the effective elastic moduli of saturated rock for low frequency, provided that the effective elastic modulus of dry rock is known in advance. The Mori–Tanaka model has the advantage of allowing the determination of effective elastic properties even for a rock whose pore space is empty. We can then obtain the effective elastic modulus of dry or saturated rock.

The approach developed by Domenico (1976) allows to take into account the phenomena of partial saturation. This author suggested replacing the compression modulus of the fluid by an equivalent modulus. Two approaches can be envisaged classically: equivalent modulus is assessed either from the average of Voigt approximation (uniform deformation hypothesis), or from the average of Reuss approximation (uniform stress hypothesis). The second proposal is necessary here since the objective is to calculate the effective elastic and acoustic properties for partially saturated rock at low frequency: the fluid pressure is uniform. The bulk modulus of fluid k_{f} is then written as

$$\frac{1}{k_{\mathrm{f}}} = \frac{S}{k_{\mathrm{w}}} + \frac{1-S}{k_{\mathrm{g}}} \tag{1}$$

where k_{w} and k_{g} are the bulk moduli for the liquid and gas, respectively; and S is the degree of water saturation. Eq. (1) has a very contrasting behavior because $k_{\mathrm{g}} << k_{\mathrm{w}}$. If $S = 1$, $k_{\mathrm{f}} = k_{\mathrm{w}}$. Instead, when a small amount of gas is present in pore space, k_{f} tends to be k_{g}.

In addition, the pore fluid increases the effective density of the rock, ρ, i.e.

$$\rho = \rho_{\mathrm{m}}(1-\phi) + \rho_{\mathrm{w}}\phi S + \rho_{\mathrm{g}}\phi(1-S) \tag{2}$$

where ρ_{m}, ρ_{w} and ρ_{g} are the densities of solid matrix, water and gas phases, respectively; and ϕ is the porosity.

In a homogeneous isotropic elastic medium, the velocities of P- and S-waves can be respectively given by (Bourbie et al., 1986):

$$V_{\mathrm{P}} = \sqrt{\frac{k + 4\mu/3}{\rho}} \tag{3}$$

$$V_{\mathrm{S}} = \sqrt{\mu/\rho} \tag{4}$$

where k and μ are the bulk and shear moduli of the medium, respectively.

According to Eqs. (3) and (4), the P-wave velocity (V_{P}) depends on the bulk and shear moduli and density of the rock, while the S-wave velocity (V_{S}) depends only on the density (ρ) of the rock and the shear modulus (μ).

The objective of this paper is to predict the elastic and acoustic behaviors of isotropic porous materials of a solid matrix containing dry, saturated and partially saturated spherical pores, using a homogenization technique based on the Mori–Tanaka model. The elastic properties of calcarenite are determined from P-wave velocity measurements. The comparisons between the predictions of Mori–Tanaka model and experimental results are analyzed.

2. Theoretical background

The homogenization method is usually used to replace a real heterogeneous medium by a fictitious homogeneous medium equivalent (Xu, 2004). We consider that a linearly elastic isotropic two-phase medium constitutes a matrix containing inclusions (Fig. 1). The elastic tensor, bulk and shear moduli of the phase i (i = matrix or inclusion) are respectively denoted by $\mathbf{C}_i$, k_i and μ_i, then the macroscopic behavior is linearly elastic and isotropic. Macroscopic stress ($\boldsymbol{\Sigma}$) and strain ($\boldsymbol{E}$) tensors are connected by

$$\boldsymbol{\Sigma} = \mathbf{C}^{\mathrm{hom}} : \boldsymbol{E} \tag{5}$$

where

$$\mathbf{C}^{\mathrm{hom}} = 3k^{\mathrm{hom}}\boldsymbol{J} + 2\mu^{\mathrm{hom}}\boldsymbol{K} \tag{6}$$

where k^{hom} and μ^{hom} denote the bulk and shear moduli homogenized, respectively; $\boldsymbol{J}$ and $\boldsymbol{K}$ represent the spherical and deviatoric tensors, respectively, and $J_{ijkl} = \delta_{ij}\delta_{kl}/3$, $K_{ijkl} = I_{ijkl} - J_{ijkl}$, $I_{ijkl} = (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})/2$, $\boldsymbol{J}:\boldsymbol{J} = \boldsymbol{J}$, $\boldsymbol{K}:\boldsymbol{K} = \boldsymbol{K}$, $\boldsymbol{J}:\boldsymbol{K} = \boldsymbol{K}:\boldsymbol{J} = 0$ (Giraud et al., 2008), where I_{ijkl} is the fourth-order symmetric identity tensor, and δ_{ij} is the Kronecker symbol ($\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 0$ otherwise).

The elastic tensors of the matrix and inclusion can be respectively written by

$$\mathbf{C}_{\mathrm{m}} = 3k_{\mathrm{m}}\boldsymbol{J} + 2\mu_{\mathrm{m}}\boldsymbol{K} \tag{7}$$

$$\mathbf{C}_{\mathrm{inc}} = 3k_{\mathrm{inc}}\boldsymbol{J} + 2\mu_{\mathrm{inc}}\boldsymbol{K} \tag{8}$$

where subscripts 'm' and 'inc' represent matrix and inclusion, respectively.

At the microscopic scale, the law of behavior at the point $\vec{r}$ is

$$\boldsymbol{\sigma}(\vec{r}) = \mathbf{C}(\vec{r}) : \boldsymbol{\varepsilon}(\vec{r}) \tag{9}$$

where $\boldsymbol{\sigma}(\vec{r})$ and $\boldsymbol{\varepsilon}(\vec{r})$ are the microscopic stress and strain tensors, respectively; and $\mathbf{C}(\vec{r})$ is the microscopic elastic tensor.

The averages of the macroscopic stress and strain are given by

$$\left.\begin{aligned} \boldsymbol{\Sigma} &= \langle \boldsymbol{\sigma}(\vec{r}) \rangle \\ \boldsymbol{E} &= \langle \boldsymbol{\varepsilon}(\vec{r}) \rangle \end{aligned}\right\} \tag{10}$$