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Rotational degree-of-freedom synthesis: An optimised finite difference method for non-exact data

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ABSTRACT

Measuring the rotational dynamic behaviour of a structure is important for many areas of dynamics such as passive vibration control, acoustics, and model updating. Specialist and dedicated equipment is often needed, unless the rotational degree-of-freedom is synthesised based upon translational data. However, this involves numerically differentiating the translational mode shapes to approximate the rotational modes, for example using a finite difference algorithm. A key challenge with this approach is choosing the measurement spacing between the data points, an issue which has often been overlooked in the published literature.
The present contribution will for the first time prove that the use of a finite difference approach can be unstable when using non-exact measured data and a small measurement spacing, for beam-like structures. Then, a generalised analytical error analysis is used to propose an optimised measurement spacing, which balances the numerical error of the finite difference equation with the propagation error from the perturbed data. The approach is demonstrated using both numerical and experimental investigations. It is shown that by obtaining a small number of test measurements it is possible to optimise the measurement accuracy, without any further assumptions on the boundary conditions of the structure.

1. Introduction

Experimental rotational degrees-of-freedom (RDOF) are required in many areas of dynamics, such as structural modification [1,2], acoustics [3], and model updating/reduction [4,5]. Whilst the measurement of translational data is now commonplace, the same cannot be said for their rotational counterparts. Although techniques exist to directly measure RDOFs, they usually require specialist equipment, such as laser vibrometers or rotational accelerometers, which may not be readily available. For this reason, synthesis methods are often used to extract rotational information form translational data, which can be measured using standard test equipment. The most common is the finite difference (FD) technique, first proposed by Sattinger in 1978 [6].

The method applies a finite difference equation to data collected from closely spaced sensors to numerically differentiate the translational data with respect to the spatial coordinate. However, as with any numerical method, its accuracy is dependent on the choice of spacing between data points. It is well documented that the accuracy of a FD equation can be improved by reducing the spacing; this paper will show that when using non-exact measured data (data containing some error), the method becomes unstable. As the spacing is decreased, small errors or perturbations in the input data, such as noise or misalignment, give rise to

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large errors in the output. Hence, a compromise must be found, which balances the numerical error of the FD equation with the perturbation propagation error from the data.

Whilst the numerical errors associated with finite difference equations are well known, little attention has been paid to the propagation error. For this reason the application of the FD method for rotational degree-of-freedom synthesis is not robust. In this paper, a full analytic error analysis of the FD method in the modal domain is carried out, showing that, for any structure, the method becomes unstable when using non-exact data. The results from the error analysis are used to propose an optimum spacing to balance the two errors. However, the optimum spacing relies on knowledge of two unknown quantities, the high order derivatives of the translational data and the error contained in the measurement. Analytical solutions for the mode shapes of beams are used to show that, for certain finite difference equations, this information can be found from the translational modal model, whilst for all other finite difference equations, a good approximation can also be found. An experimental investigation is also carried out to show how effective the method can be when approximating the measurement error.

The applications of experimentally derived rotational degrees-of-freedom are vast and varied. Schmitz et al. [1,2] proposed a structural modification method, called receptance coupling substructure analysis (RCSA), as a method to avoid chatter in machining operations such as milling. Moorhouse et al. [3] used the finite difference method in order to characterise structure-borne sound sources for use in assembled structures, such as vehicles and machinery. And the Craig-Brampton method [5] (more commonly known today as component mode synthesis) was originally proposed as a model reduction method. Whilst these applications are promising, they are all limited by the need for highly accurate information on the rotational degrees-of freedom. As stated above, this paper concentrates on the FD method, partly due to its simplicity, but also due to the lack of robust error analysis available for the method.

Sattinger first considered the problem of rotational degree-of-freedom synthesis in 1980 [6], showing that any rotational frequency response function (FRF) is equal to the spatial derivative of its translational equivalent. The finite difference method was then used to approximate such derivatives. Using a free-free beam as an example, it was found that the results were accurate close to resonance, whilst other areas of the FRF showed considerable scatter. Although it was shown (using theoretical data) that a smaller spacing increases the accuracy of the numerical method, the link between increasing the spacing and decreasing the scatter was not made. Sestieri et al. [7] also used the example of a free-free beam, this time with experimental data. They suggested an improvement in the result could be found by using a spacing of between 5% and 8% of the total beam length; however, they failed to recognise that the error level may vary between different experiments and with different beams.

Duarte and Ewins [8,9] later looked at the same problem and had similar issues, noting that the spacing of the accelerometers affects not only the scatter in the results but also the position of the antiresonances. The paper was also the first to apply the finite difference method in the modal domain. Again using data from a free-free beam, rotational mode shapes were approximated by applying the FD equation directly to the measured mode shapes, and then the rotational FRFs constructed from the result. This was found to give more accurate results than application in the frequency domain, but carried the added difficulty of how to include high frequency residuals. A high frequency pseudo mode was found to give satisfactory results. Although the paper suggests that the quality of the result is directly linked to the spacing, it is also concluded that the selection of the appropriate spacing remains a problem.

The only theoretical error analysis of the finite difference method for use in rotational DOF synthesis came from Elliot [10,11]. In this case, using a simply supported beam as an example, it was shown that the numerical error associated with FD equations is directly proportional to the spacing between the sensors. However, the paper does not go on to show that the perturbation propagation error is inversely proportional to the spacing; instead arguing that, due to improvements in measurement equipment/practice, this should be less of a problem.

2. Background

The translational displacement of a structure can be characterised by its mode shapes $\phi_r(x)$ and their corresponding eigenvalues ω_r^2, which are easily related to the frequency response function (FRF), H_{ij}, usually measured in modal testing.

$$H_{ij}(\omega) = \frac{Y_i(\omega)}{F_j(\omega)} = \sum_{r=1}^{N} \frac{\phi_r(x_i)\phi_r(x_j)}{\omega_r^2 - \omega^2} \tag{1}$$

Here, $H_{ij}(\omega)$ is the FRF excited at location i and measured at location j for a particular frequency ω, Y_i is the Fourier transform of the displacement at location i, F_j is the Fourier transform of the input force at location j, N is the total number of modes measured, $\phi_r(x_i)$ is the *rth* mode at location i, and the magnitude of ω_r is the *rth* natural frequency.

However, the above equation only makes up part of the full response model often required for the applications discussed in section 1, as it only considers the translational displacement, Y, and excitation force, F. To fully understand the vibration of any structure both rotational displacement Θ and excitation moment M must also be included, giving rise to three further FRFs:

$$N_{ij}(\omega) = \frac{\Theta_i}{F_j} = \sum_{r=1}^{N} \frac{\phi_r^{(1)}(x_i)\phi_r(x_j)}{\omega_r^2 - \omega^2} \qquad L_{ij}(\omega) = \frac{Y_i}{M_j} = \sum_{r=1}^{N} \frac{\phi_r(x_i)\phi_r^{(1)}(x_j)}{\omega_r^2 - \omega^2} \qquad P_{ij}(\omega) = \frac{\Theta_i}{M_j} = \sum_{r=1}^{N} \frac{\phi_r^{(1)}(x_i)\phi_r^{(1)}(x_j)}{\omega_r^2 - \omega^2} \tag{2}$$

Whilst the measurement of these FRFs is difficult, mainly due to the application of a pure moment, Eq. (2) shows that they can be constructed from the standard modal model (ϕ_r, ω_r) and the rotational mode shapes $\phi_r^{(1)}$ (where the superscript (1) represents