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# Experimental identification of closely spaced modes using NExT-ERA

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This article presents a study on the capability of the time domain OMA method, NExT-ERA, to identify closely spaced structural dynamic modes. A survey in the literature reveals that few experimental studies have been conducted on the effectiveness of the NExT-ERA methodology in case of closely spaced modes specifically. In this paper we present the formulation for NExT-ERA. This formulation is then implemented in an algorithm and a code, developed in house to identify the modal parameters of different systems using their generated time history data. Some numerical models are firstly investigated to validate the code. Two different case studies involving a plate with closely spaced modes and a pulley ring with greater extent of closeness in repeated modes are presented. Both structures are excited by random impulses under the laboratory condition. The resulting time response acceleration data are then used as input in the developed code to extract modal parameters of the structures. The accuracy of the results is checked against those obtained from experimental tests.



## 1. Introduction

Repeated modes or closely spaced modes which occur in structures such as circular plates, gears or bladed-discs that have some degrees of symmetry. From an experimental point of view, closely spaced modes are two close modes that are sometimes misinterpreted as one single mode. Failure in identification of closely spaced modes, which are normally associated with pseudo-repeated root problem [1], during dynamic analysis may lead to critical effects in dynamic systems such as pogo-effect in Titan 2 space rocket [2]. Therefore the possibility of occurrence of closely spaced modes during modal analysis process is inevitable. Moreover, in model updating process one needs an experimental mode corresponding to each of those identified analytically. This requirement dictates that all the existing modes, including those which are closely spaced, have to be extracted employing more adapted techniques.

During the last four decades different algorithms have been proposed to extract modal parameters of structures by researchers. These algorithms, which are basically developed in time, frequency, or space domains, are categorized into single input-single output, single input-multi output or Multi Input-Multi Output (MIMO) methods. Among these, frequency and time domains are considered to be more practical. Time domain methods which directly use experimentally recorded time

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history data avoid some of the problems (such as leakage) which are normally associated with the Fourier transform in the frequency domain methods. On the other hand, in time domain methods a reasonable signal to noise ratio suffices to extract modal properties without the necessity of measuring excitation force functions [3]. Avoiding FRF calculations in time domain algorithms together with its superior damping estimation [4] makes these methods more suitable to analyze closely spaced modes of structures. However, some frequency domain methods have been successfully employed to extract the closely spaced modes of structures [5–7].

As reported in Ref. [8], taking into account the response at different DOFs, MIMO procedures are better candidates to analyze complex structures with closely spaced or even repeated modes. In 1982, Vold et al. [9] proposed Polyreference Complex Exponential (PRCE) method which is considered as the first developed MIMO algorithm. Two other well-known time domain MIMO modal identification techniques which were developed later are the Extended Ibrahim Time Domain (EITD) [10] and Eigensystem Realization Algorithm (ERA) [11].

It has been reported that Experimental Modal Analysis (EMA) methods, even the MIMO ones, face difficulty when dealing with close modes [5]. Because in these methods due to the point excitation, the identification of closely spaced modes highly depends on the excitation technique and the excited region. Moreover, the methods have shown inconsistency in calculating precise damping ratios when it comes to close modes. While, unlike methods in EMA, due to the broadband random excitation in real loading situation, which acts through different points of the structure, OMA methods can excite all vibration modes. Additionally, because of their MIMO nature, the OMA methods are more accurate in studying the closely spaced modes of structures.

As mentioned earlier, in time domain methods there is no need to record excitation data. This notion, accompanied with the fact that Operational Modal Analysis (OMA) only requires output data due to natural or ambient loading, was the motivation in this study to employ OMA in conjunction with time domain schemes. For modal identification of structures, OMA methods may be employed for the whole structure at once without any need to elaborate excitation equipment.

In 1992 the Natural Excitation Technique (NExT) was introduced [12] to process the random response data of the structure, so that the results may be used as an input to MIMO-type time domain EMA algorithms such as EITD, ERA and PRCE. Indeed, NExT in combination with any MIMO time domain algorithm can be considered as an OMA method in its own right. For example Caicedo [13,14] used NExT-ERA to extract the modal parameters of four-story steel structures.

Among all the time domain methods in modal analysis, ERA has proved to have great accuracy and is considered as one of the most robust and successful methodologies for engineering practices [15–17]. Several studies have also been conducted to evaluate the accuracy of the procedure for various systems, as well. Siringoringo [17] exploited the method for system identification of a suspension bridge. Chiang and Lin [18] studied the accuracy of a modified version of ERA method for coupled simulated mass spring systems. Zhang et al. [19] used the methodology for extracting the modal properties of a bridge model. The method has also proved useful for structural health monitoring purposes [13,20].

Moreover, many researches have been conducted to study the effect of close modes on performance of different methods in modal analysis. The efficacy of FDD [5] was investigated in the case of close modes by Brinker et al., just when it was developed. and Agneni [21] extracted the close modes of a AB-204 helicopter blade using Hilbert transform method and frequency domain decomposition. Ranieri [22] also studied the effectiveness of OMA time domain method and second order blind identification in case of close modes. Brincker et al. [23] also introduced a modified version of EITD to extract repeated modes of numerical structures using output data only. Chen [24] studied the efficiency of the analytical mode decomposition method in case of closely-spaced modes, as well. The concept of closely-spaced modes has proved so critical today that every new system identification technique is evaluated in the case of close modes just when it is proposed [6,7,25,26]. NExT-ERA has also been employed for identification of closely-spaced modes of structures. Caicedo [14] managed to extract the low-frequency close modes of a benchmark structure by NExT-ERA. In Siringoringo's study [17] the method was also employed for system identification of a suspension bridge. In his study, the structure had some low frequency closely-spaced natural frequencies which were successfully identified by NExT-ERA.

This research aims to assess the accuracy of the time domain method ERA in corporation with NExT for identification of close modes in higher frequencies, where noise is a more critical issue and makes the identification process more difficult. Moreover, for symmetric plate-like structures there is some difficulty in identifying accurate mode-shapes [28], the NExT-ERA method is also evaluated in such a case, as well. For this purpose, first a simulated mass-spring system and a numerically modeled beam are studied to verify the procedure and formulation. The effectiveness of the method is then evaluated for a plate having closely spaced modes. Finally two separate experiments are carried out on a plate and a pulley ring with closely spaced modes and the effectiveness of the method is studied experimentally.

## 2. Eigensystem Realization Algorithm

ERA was developed by Juang and Pappa [11] based on the concepts that predominantly originated form the control theory, so that it differs from the usual developments found in the EMA literature. In ERA technique, using dynamic equation of equilibrium for an N-DOF viscously damped system together with the state vector $\{\mathbf{u}(t)\} = \{\mathbf{y}(t),\ \dot{\mathbf{y}}(t)\}$, in which $\mathbf{y}(t)$ is displacement vector, the output of the system at $k$th time sample is written as $\{\mathbf{x}(k)\} = [\mathbf{R}]\{\mathbf{u}(k)\}$, where $\{\mathbf{u}(k)\} = [\mathbf{A}]\{\mathbf{u}(k-1)\} + [\mathbf{B}]\{\delta(k-1)\}$, $\delta(k)$ is input vector at $q$ locations and $[\mathbf{A}]$ and $[\mathbf{B}]$ are discrete-time state-space matrices which are determined in a process called realization. $\{\mathbf{x}(k)\}$ is a vector of dimension $p$ which is the number of output