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Non-conservative stability of spinning pretwisted cantilever beams

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ABSTRACT

The stability of a pretwisted cantilever beam spinning about its longitudinal axis and subjected to non-conservative force is investigated. In this study, it is assumed that the cantilever is embedded in viscoelastic medium, which is modeled by the Kelvin-Voigt foundation. Two different types of the non-conservative force are considered. The governing equations of motion and boundary conditions are derived by using Hamilton's principle. The finite element method is utilized to transform the coupled equations of motion to a general eigenvalue problem. The proposed model is justified by an excellent agreement between the present results and those reported in the literature. The effects of several design parameters including the pretwist angle, the cross section ratio, the viscoelastic parameters and load span length on the stability of the spinning pretwisted cantilevers are also examined. Moreover, the critical load and spinning speed and stability regions of the spinning cantilevers are identified. The results show that the design parameters significantly change the stability of the spinning pretwisted cantilever beams.

1. Introduction

The stability analysis of mechanical elements, such as beams, plates and shells, is an important task for structural design and it has been attracted a considerable attention within the scientific and engineering communities [1,2]. Among different types of the structural elements, pretwisted beams are widely used in numerous engineering applications [3]. One of the most familiar types of the pretwisted beams is drill [4]. During drilling and milling process, the drills are always subjected to different compressive loads [5]. Therefore, the stability analysis of these pretwisted beams has been attracted extensive attention. Furthermore, the dynamic instability (or flutter) in an operating the drill bits, the drill-rods and the drill-strings was experimentally observed by different researchers [6–9]. Kar and coworkers [9–12] studied the dynamic stability of different pretwisted cantilever beams subjected to follower forces. The dynamic stability of the cantilever pretwisted beams subjected to different types of non-conservative force was studied by Celep [13,14]. The stability of pre-twisted beam subjected to a pulsating axial force with various boundary conditions was investigated by Gürgöze [15]. In another study, Mohanty [16] studied the effects of an axial compressive load and a localized damage on the dynamic stability of a pretwisted cantilever beam. Recently, Abed et al. [17] investigated the elastic buckling of pretwisted columns by using the linear perturbation method. More recently, the dynamic stability of a pretwisted cantilever beam subjected to a distributed follower force was

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studied by Fazelzadeh et al. [18]. The mentioned studies indicated that the stability of the pretwisted beams is strongly affected by the aspect ratio, the pretwist angle and types of applied load.

As the pretwisted beam spins about its axial direction, the spinning speed plays a significant role and has to be taken into consideration. The elastic stability of a spinning pretwisted beam subjected to a conservative axial load was conducted by Liao and Dang [19], for the first time. Effects of the spinning speed, the pretwist angle and the aspect ratio of the spinning pre-twisted beams on their stability were investigated in Refs. [20,21] using the Euler-Bernoulli beam theory and the assumed mode method. On the basis of the finite element method, Liao and Huang [22] studied the parametric instability of a spinning pretwisted beam under a periodic end axial force. Influences of the axial compressive load and the spinning speed on the stability of a pretwisted cantilever, as a simple model for a drill bit, were discussed by Tan et al. [23,24] on the basis of the Euler-Bernoulli beam theory. To investigate the stability of the spinning pretwisted beams subjected to axial random forces, Young and Gau [25] extended the work of Liao and Huang [22]. For numerical calculations, they used the finite element method and the mean-square stability criterion to determine the stability boundary of the pretwisted beam. In another study [26], the dynamic stability of the spinning pretwisted beam with non-constant spin rates and under an axial random force was investigated on the basis of the finite element method and the stochastic average method. Effect of periodic drilling force on the dynamic instability of the drill bit during drilling process was investigated by Huang et al. [27]. Their results showed that the instability of the drill bit was affected by the applied force, the spinning speed and the pretwist angle. Using the finite element method, Lin and Chen [28] studied the dynamic stability problems of the spinning pretwisted sandwich beams subjected to periodic axial loads. The stability of the spinning pretwisted Timoshenko beams subjected to axial compressive loads was studied by Chen [29,30] using the finite element method.

The above discussion reveals that few investigations have been performed previously on the stability of the spinning pretwisted cantilever beams subjected to the non-conservative loads. Generally, several types of the non-conservative loads may be applied to the structures such as time and configuration dependent loads. The configuration dependent load, the so-called follower force, is one of the most interesting types of the non-conservative loads [31–33]. The follower forces are generally classified as concentrated follower force, distributed follower force and partially distributed follower force. One example of the non-conservative follower force is compressive force acting on the drills during the drilling process [34]. This point may be explained as follows. During the drilling process, the drill bit is subjected to a friction force acting on the edge of the drill tool. This force changes its direction with the cantilever deflection during the operation. Therefore, the load can be considered as a lively load or the follower force. Hence, to develop a comprehensive model to study the stability of the spinning pretwisted cantilever beams, it is necessary to consider different types of the follower force.

In this research, the stability of the spinning pretwisted cantilever beams subjected the non-conservative follower forces is investigated. In order to cover a wide range of possible application for the spinning pretwisted cantilever beams, two different types of the follower force are considered. The first type is the concentrated follower force acts at the free end of the cantilever and the second type is the partially distributed follower force, which exerted along the longitudinal axis of the cantilever. Furthermore, the drill bit during the drilling process is under the effect of a work piece [35]. To include this constriction effect, the viscoelastic medium is employed in this study. The viscoelastic medium is modeled as the Kelvin–Voigt foundation. The equations of motion are derived by using Hamilton's principle and the finite element method is utilized to solve the problem. To demonstrate the accuracy of the proposed model, the numerical results are compared with those reported in the literature. Furthermore, the effects of several design parameters are elucidated. The present study may provide useful information concerning the stability of the drill during the drilling process.

2. Problem statement

Spinning pretwisted cantilever beams with length L, mass per unit length ρA and Young's modulus E, and the corresponding appropriate coordinate system are shown in Fig. 1 for two types of the follower force; (a) a tip follower force and (b) partially distributed follower force. It is assumed that the cantilever spins about its longitudinal axis with a constant rate Ω^*. The cross section of the cantilever rotates linearly about the longitudinal axis of the beam with a total pretwist angle α. Every cross section of the cantilever beam is symmetric with respect to the two principal axes of inertia. Therefore, it can be assumed that there is no coupling between bending and torsional motions for the pretwisted beam. In addition, flexural bending takes place simultaneously in two mutually perpendicular planes [25]. In this study, it is assumed that the spinning cantilever works in the viscoelastic medium with the total reaction force per unit length of $\mathbf{f}_{\mathbf{v.m.}}(x,t) = f_y(x,t)\mathbf{j} + f_z(x,t)\mathbf{k}$. As it can be seen from Fig. 1b, in the case of the distributed follower force, X_s represents the length of the cantilever embedded in the viscoelastic medium and subjected to the follower force.

Here, two coordinate systems will be utilized (Fig. 2). In this figure, (x, y, z) with $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ unit vectors is a fixed coordinate system, while (X, Y, Z) is a rotating coordinate system attached to the cantilever cross section in the deformed state. Furthermore, η and ξ axes coincide with the principal axes of the pretwisted beam at every cross section with the pretwist angle of $\varphi(x) = \alpha x/L$. In addition, $v(x,t)$ and $w(x,t)$ represent the transverse displacements of the cantilever along the Y- and Z-axes, respectively.