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Exact nonstationary responses of rectangular thin plate on Pasternak foundation excited by stochastic moving loads

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ABSTRACT

This paper develops an efficient method termed as PE-PIM to address the exact nonstationary responses of pavement structure, which is modeled as a rectangular thin plate resting on bi-parametric Pasternak elastic foundation subjected to stochastic moving loads with constant acceleration. Firstly, analytical power spectral density (PSD) functions of random responses for thin plate are derived by integrating pseudo excitation method (PEM) with Duhamel's integral. Based on PEM, the new equivalent von Mises stress (NEVMS) is proposed, whose PSD function contains all cross-PSD functions between stress components. Then, the PE-PIM that combines the PEM with precise integration method (PIM) is presented to achieve efficiently stochastic responses of the plate by replacing Duhamel's integral with the PIM. Moreover, the semi-analytical Monte Carlo simulation is employed to verify the computational results of the developed PE-PIM. Finally, numerical examples demonstrate the high accuracy and efficiency of PE-PIM for nonstationary random vibration analysis. The effects of velocity and acceleration of moving load, boundary conditions of the plate and foundation stiffness on the deflection and NEVMS responses are scrutinized.

1. Introduction

The dynamic response analyses of pavement structures of highway bridges, roadways and airport runways acted by vehicle loads are paramount during their design phase. In the past decades, the dynamic analyses of the pavement structures subjected to deterministic moving loads were widely investigated [1–3].

Generally, the pavement structure is modeled as a beam, plate or those resting on an elastic foundation. Frýba [1] reviewed the previous studies on the forced vibration of beam and plate structures under deterministic moving load, and obtained some analytical solutions by using Fourier and Laplace transform. Actually, the researches on the analytical solutions of pavement structures to moving loads were involved separately in the infinite and finite length models. In general, the integral transform methods such as Fourier transform and Laplace transform are applied for the infinite length model [4–10]. For the finite length model, however, normal modal method is suitable to achieve the analytical dynamic responses. By virtue of normal modal method and analytical Duhamel's integral, Hilal and Zibdeh [11] and Li et al. [12] derived the closed-form

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solutions of finite length beam and plate structures subjected to accelerating, decelerating and constant velocity moving load. However, the analytical solutions can be achieved only for a few specific cases. In practice, the semi-analytical approaches are adopted usually for many cases, e.g., the fast Fourier transform can substitute for the analytical Fourier transform [13–15], and the time integration algorithms replace the analytical Duhamel's integral [16,17]. Moreover, the fully numerical methods were also employed to address this kind of problem, even including geometrically nonlinear case [18–20]. Huang and Thambiratnam [18,19] investigated the dynamic responses of a rectangular plate resting on Winkler elastic foundation under accelerated moving load with the finite strip method. Their results are also used to verify the proposed method in this work.

In aforementioned studies, the Winkler elastic foundation model is popularly utilized to describe the mechanical behavior of soil in the pavement system due to its simplicity. In fact, this model cannot reflect truly the continuity of soil. The importance of horizontal resistance in elastic foundation was stressed by Lu et al. [15] and Kim [21]. Additionally, Kargarnovin and Younesian [22] examined the dynamic responses of Timoshenko beams on Pasternak foundation with two parameters under moving load by integral transform method. This paper focuses on the random vibration of finite length rectangular thin plate resting on bi-parametric Pasternak foundation, which is a general model capable of considering well the continuity of soil and horizontal resistance in an elastic foundation subjected to stochastic moving loads.

The action on pavements by the vehicles can be modeled by a deterministic moving force or a moving mass [23]. The latter accounts for the inertia effect which can result in the dynamic system to be time-varying, and bring a challenge in achieving the exact solutions, as reviewed by Ouyang [2]. In fact, the inertia force acted on the pavement structure is stochastically varying with time due to the roughness of the pavement surface. To simplify the analysis, the resultant force imposed by stochastic inertia force and the gravity of the vehicles can be tackled as a stochastic load acting on the pavement structure. Frýba [24] investigated earlier the random vibration of beam subjected to stochastic moving load, and adopted the correlation function method in time domain to obtain the mean square responses, as well as pointed out the stochastic responses are nonstationary even if the loads are stationary random process due to the location of moving load varying with time. Similarly, Zibdeh et al. [25,26] analyzed random vibration responses of Rayleigh, Timoshenko and laminated composite beams with general boundary conditions under varying velocity moving load. However, their analytical solutions are approximated by ignoring the modal cross-correlation terms. By virtue of Galerkin's method and Perturbation technique, Chang et al. [27] examined dynamic responses of the Bernoulli-Euler beam on Winkler foundation under moving mass with uncertain parameters such as mass, damping, velocity and acceleration. Rystwej and Sniady [28] tackled the random vibration problem of infinite length beam and plate on Pasternak foundation by assuming the moving load as a Poisson process, and calculated the various moments of responses. Yin et al. [29] carried out the nonstationary random vibration analysis of bridge structure with introducing the power spectral density (PSD) of road roughness in space domain. Nevertheless, their works belong to numerical simulation, which may be performed to obtain the statistical property of responses. Khavassefat et al. [30] investigated the nonstationary stochastic responses of the pavement in frequency-wavenumber domain with integral transform, based on field measurements data of the surface roughness. In frequency domain analysis, the power spectral method can suitably replace the correlation function method, when the amplitude of moving load is assumed as a Gaussian process. In the framework of power spectral analysis for discretized model with multiple degrees of freedom, Lin et al. [31] proposed a highly efficient and accurate algorithm named pseudo excitation method (PEM), which can transform the nonstationary random vibration analysis of structure into deterministic time history analysis, and extended the PEM to the coupled rail system subjected to stochastic moving load [32,33]. In this paper, PEM is also employed to address the exact solutions of nonstationary stochastic responses of the plate on Pasternak elastic foundation based on continuous model. Nevertheless, since the PEM is just applicable to deal with the time-invariant system currently, the stochastic inertia force and the gravity of the vehicles are treated as a Gaussian stationary stochastic process.

In dynamic analysis of the pavement system, apart from the deflection response, the predication of stress distribution in the pavement structure is also indispensable. According to Kirchhoff thin plate theory, there exist three stress components, i.e., a couple of normal stress and one shear stress. In deterministic dynamic analysis, an important evaluation index is the von Mises stress [34]. The root mean square (RMS) of the von Mises stress process of rail structure to random stationary excitation is calculated in Refs. [35,36]. In stochastic dynamic analysis, however, the von Mises stress, a nonlinear function of the stress components, is a constantly positive non-Gaussian process containing non-zero mean and inconsistent frequency contents (i.e., peak PSD frequencies) with that of the stress components. Preumont and Piefort [37] suggested a Gaussian equivalent von Mises stress process with zero-mean to replace the von Mises stress process for estimating the fatigue life of the structure. Nevertheless, it does not contain the cross-correlation terms of normal and shear stress [38]. Bonte et al. [39] proposed another equivalent von Mises stress, whose PSD formula takes into account the phase differences and just contains the cross-PSD functions between one of the normal stress components and the shear stress component.

The objective of this work is to develop the efficient method namely PE-PIM, which combines PEM with precise integration method (PIM), for exact stochastic response analysis of thin plate resting on Pasternak foundation subjected to accelerated moving stochastic load. Accordingly, the rectangular plates with simply supports along the edges of the load starting position and terminating position are considered, owing to the existence of the exact solutions of free vibration for these cases. Another purpose is to investigate the distribution of the stochastic stress in the plate. To this end, the new equivalent von Mises stress (NEVMS) as a Gaussian process whose PSD function contains all cross-correlation terms between stress components is proposed based on the PEM.

This paper is organized as follows. Section 2 presents the differential equation of motion for rectangular thin plate with Pasternak elastic foundation and classical boundary conditions, and the exact Lévy solutions of free vibration [40] are also