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Vibrations of an Euler-Bernoulli beam with hysteretic damping arising from dispersed frictional microcracks

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ABSTRACT

We study free and harmonically forced vibrations of an Euler-Bernoulli beam with rate-independent hysteretic dissipation. The dissipation follows a model proposed elsewhere for materials with randomly dispersed frictional microcracks. The virtual work of distributed dissipative moments is approximated using Gaussian quadrature, yielding a few discrete internal hysteretic states. Lagrange's equations are obtained for the modal coordinates. Differential equations for the modal coordinates and internal states are integrated together. Free vibrations decay exponentially when a single mode dominates. With multiple modes active, higher modes initially decay rapidly while lower modes decay relatively slowly. Subsequently, lower modes show their own characteristic modal damping, while small amplitude higher modes show more erratic decay. Large dissipation, for the adopted model, leads mathematically to fast and damped oscillations in the limit, unlike viscously overdamped systems. Next, harmonically forced, lightly damped responses of the beam are studied using both a slow frequency sweep and a shooting-method based search for periodic solutions along with numerical continuation. Shooting method and frequency sweep results match for large ranges of frequency. The shooting method struggles near resonances, where internal states collapse into lower dimensional behavior and Newton-Raphson iterations fail. Near the primary resonances, simple numerically-aided harmonic balance gives excellent results. Insights are also obtained into the harmonic content of secondary resonances.

1. Introduction

Vibrational energy dissipation in many materials is approximately rate independent and hysteretic; for some classical references, see Refs. [1–3]. In particular, even though amplitudes of free vibrations might decay exponentially, thereby suggesting linear damping models, the actual damping may well be hysteretic and nonlinear. Several models of scalar hysteresis [4–9] have been proposed in the literature. These are useful for lumped parameter or low dimensional systems. Generally speaking, models based directly on underlying dissipative mechanics tend to be complicated, while simpler and lower-dimensional models tend

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to be empirical.

In this paper we study the dynamic response of beams whose internal dissipation can be modeled as the net averaged effect of a large number of randomly dispersed frictional microcracks [9]. The relevant *scalar* hysteresis model has been proposed recently elsewhere [9], and our application to a vibrating beam as presented here offers useful physical insights into the behavior of the beam for both free and harmonically forced vibrations, as well as a clear presentation of some procedural aspects involving computation of virtual work *via* a domain integral that is approximated using a modest number of Gauss points; we have not noted a discussion of this approach in the literature.

In the rest of this section, we begin with a brief review of the relevant literature on modeling of hysteretic dissipation, and then present the main motivation for our paper.

Several authors have incorporated overall hysteretic dissipation without explicitly modeling the rate-independent hysteresis at any stage. In 1974, Lund [10] used a hysteretic loss factor, based on the phase difference between stress and strain due to hysteresis, for the analysis of a flexible rotor supported by fluid-film bearings. In 1977, Nelson and Zorzi [11] applied a similar loss factor in the finite element formulation of an internally damped rotor bearing system. In 1990, Lee et al. [12] used a complex Young's modulus to incorporate the hysteretic energy loss in a non-uniform Euler-Bernoulli cantilever beam. The same concept was used by Chang [13] and Gounaris et al. [14] for the vibration analysis of structures. Later on, Labonnote et al. [15] used complex values for both elastic and shear moduli for Timoshenko timber beams.

Some papers have explicitly incorporated approximate models for hysteretic stress strain relations. For example, Baker et al. [16] implemented a hysteretic stress strain model proposed by Pisarenko [17] to study the free vibration of a cantilever beam. In this approach, increasing and decreasing portions of the stress strain curve are separately and explicitly described using quadratic approximations designed to match desired per-cycle dissipation rates. This method is effectively restricted to simple load cycles without small reversals within load paths (we will discuss this issue further in the context of our own model below). Inman [18] studied a viscoelastic beam incorporating hysteretic stress strain behavior using a linearized convolution operator attributed to Christensen [19]. These approaches provide examples of approximating the within-cycle hysteretic behavior to make semi-analytical or numerical progress easier.

Some other papers present direct implementations of scalar hysteresis models. For instance, Papakonstantinou et al. [20] and Gkimousis and Koumousis [21] used the Bouc-Wen model [5,6] to explicitly incorporate hysteretic dissipation in their structural responses. In this approach, governing equations for the visible external displacement variables and the hidden internal hysteretic variables are to be numerically solved together in forward simulation. Similar explicit solution of additional evolution equations governing hysteretic quantities may be found in Refs. [22–24]. Such explicit numerical solution of hysteresis equations is common to our own approach below, although our hysteresis model has a different physical motivation.

Some authors have modified or extended the Bouc-Wen model to enable better matches with data, or better shape control (e.g., pinching or degradation): see, e.g., [25–27].

Among alternatives to the Bouc-Wen model, we note Segalman's [28] four parameter hysteresis model based on the earlier model of Iwan [4], with the aim of simulating the dissipative behavior of joints. In a loosely related approach, Quinn and Segalman [29] developed a model using Jenkins elements in series to describe the microslip induced dissipation of a mechanical joint. Song et al. [30] used an adjusted Iwan model to depict the damping in a beam structure with bolted joints, with neural networks to fit the Iwan model parameters.

The foregoing review of hysteresis models and their use is indicative, but not complete. A few more noteworthy papers which differ slightly in their aims are now mentioned. Dahl [31] developed an early hysteresis model to simulate solid friction which shares some features with the Bouc-Wen model but has not become as popular. Another model by Valanis [32] was used by Gaul and Lenz [33] to develop a lumped parameter model for nonlinear substructures used in finite element analysis of lightweight space structures. Sivaselvan and Reinhorn [34] attempted a unifying discussion of hysteresis models with strength and stiffness degradation. In even more detailed modeling of concrete structures at a global level, with loading and unloading, yielding and degradation, and other complicated phenomena, Miramontes et al. [35] proposed a piecewise-linear moment-curvature model with various rules for different parts of load cycles. The modeling approaches mentioned in this paragraph are significantly more complicated than the hysteretic damping model we will adopt below; our relatively simpler modeling will yield some interesting new insights.

Our adopted hysteresis model is from the work of Biswas et al. [9], who performed finite element analyses of a plate with distributed frictional micro-cracks under biaxial far-field in-plane loading, and obtained a simple yet qualitatively realistic scalar hysteresis model therefrom. In their study, the far-field tractions on the plate were derived from a spatially constant stress state $\underset{\sim}{\sigma}$ multiplied by a function of time, $q(t)$, with the result that the state of zero stress was achieved whenever $q(t)$ passed through zero. The spatially constant nature of the stress led to essentially a scalar model. The resulting hysteresis loops were pinched at the origin as expected (see, e.g., [9] and also [36]). Based on the overall qualitative features of the finite element results, the following scalar hysteresis model was proposed in Ref. [9]

$$\dot{\theta} = \frac{\kappa}{(|\chi| + \epsilon)} \{\theta_a + \beta \,\mathrm{sgn}(\chi\dot{\chi}) - \theta\} |\dot{\chi}|, \quad f = \theta\chi \tag{1}$$

where κ, β, θ_a and ϵ are some positive parameters (the original article's θ_m and K are here θ_a and κ respectively to avoid some confusion later); and where χ, θ and f are the input, internal variable and output variable respectively. The parameters κ, β and θ_a in Eq. (1) control the size and the orientation of the hysteresis loop; and ε is a small regularization parameter used to relieve the singularity at $\chi = 0$. A typical response of the model (Eq. (1)) under a multi-frequency input is shown in Fig. 1. The presence of minor loops shows that the model can approximately capture partial unloading effects, which are not captured by the usual