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An efficient algorithm for the retarded time equation for noise from rotating sources

S. Loiodice [a], D. Drikakis [b,*], A. Kokkalis [c]

[a] Barclays, London, E14 5HP, United Kingdom
[b] University of Strathclyde, Glasgow, G1 1XJ, United Kingdom
[c] Hellenic Air Force Academy, Attica, Greece

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ABSTRACT

This study concerns modelling of noise emanating from rotating sources such as helicopter rotors. We present an accurate and efficient algorithm for the solution of the retarded time equation, which can be used both in subsonic and supersonic flow regimes. A novel approach for the search of the roots of the retarded time function was developed based on considerations of the kinematics of rotating sources and of the bifurcation analysis of the retarded time function. It is shown that the proposed algorithm is faster than the classical Newton and Brent methods, especially in the presence of sources rotating supersonically.

1. Introduction

The solution of the retarded time equation is the first requirement in any noise prediction code, independently of the implemented formulation. In order to better understand this statement it is helpful to describe briefly the most common formulations for the prediction of noise from moving sources.

These formulations are based on the Ffowcs Williams - Hawkings (FW-H) equation, a generalised form of the Lighthill's Acoustic Analogy, which can be used for arbitrarily moving bodies [1]. Using this approach the noise prediction is divided into two phases: noise generation and noise propagation. The first phase is obtained via the solution of the flow field inside the dynamic source region using Computational Fluid Dynamics codes, then the noise waves are propagated, using the integral form of the FW-H equation. The propagation starts either from the body surface or from the boundary of the near-field domain, in the case of porous formulation, and reaches the observer in the far-field. This allows to save computational time and resources because the CFD solution is limited only to the near-field domain.

The main advantage of this approach with respect to the Kirchoff equation is that the results obtained using the FW-H equation are much less sensitive to the positioning of the near-field domain boundary as well as to the non-linearities in the flow field [2,3].

The FW-H equation is expressed in equation (1):

* Corresponding author. University of Strathclyde, James Weir Bld, JW6.04, 75 Montrose Str., Glasgow, G1 1XJ, United Kingdom.
E-mail address: dimitris.drikakis@strath.ac.uk (D. Drikakis).

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$$c_0^2\hbar^2 p' = \frac{\overline{\partial}^2}{\partial x_i x_j}\left[T_{ij}H(f)\right] - \frac{\overline{\partial}}{\partial x_i}\left\{\left[\Delta P_{ij}n_j + \rho u_i(u_n - \upsilon_n)\right]\delta(f)\right\} + \frac{\overline{\partial}}{\partial t}\left\{\left[\rho_0\upsilon_n + \rho\left(u_n - \upsilon_n\right)\right]\delta(f)\right\} \tag{1}$$

where $\hbar^2 = 1/c_0^2\,\partial^2/\partial t^2 - \nabla^2$ is the wave operator; $T_{ij} = \left(\rho\upsilon_i\upsilon_j + P_{ij} - c_0^2(\rho - \rho_0)\delta_{ij}\right)$ is the Lighthill's tensor, where $P_{ij} = \tau_{ij} + p\delta_{ij}$ and $\Delta P_{ij} = P_{ij} - p_0\delta_{ij}$; ρ is the density; $p' = p - p_0$ is the acoustic pressure; p and p_0 stand for pressure and pressure of the undisturbed medium, respectively; c_0 is the speed of sound; u_n is the fluid velocity normal to a surface S, which is considered permeable, and υ_n is the perturbation velocity normal to S; $\delta(f)$ is the dirac delta; and $H(f)$ is the Heaviside function; finally, the function f represents the equation of the moving surface source, which can be either a solid or a porous surface (see Ref. [4] for further details).

Equation (1) is a generalised form of the Lighthill's equation, $c_0^2\hbar^2\rho' = T_{ij}$, and this is evident from the lines over the differential operators, in Equation (1), which imply that the derivatives must be considered as generalised derivatives [5,6], and the presence of two additional terms on the right hand side of Equation (1). The three terms on the R.H.S. are known respectively as Quadrupole, Dipole or Loading and Monopole or Thickness term. Each one of these takes into account the different contributions of a complex noise source such as a helicopter rotor blade, e.g. the Thickness terms accounts for the noise generated by the blade's thickness. The monopole and dipole terms correspond to the physical contents when the FW-H surface coincides with the blade surface.

Equation (1) is effectively a generalised inhomogeneous wave equation. Studying the solution of a simplified version of this kind of equations it is possible to obtain a solution for each term of Equation (1). For instance, given the Equation: $\hbar^2\Phi(\mathbf{x},t) = Q(\mathbf{x},t)\delta(f)$, for the generic variable $\Phi(\mathbf{x},t)$ and arbitrary surface source distribution $Q(\mathbf{x},t)\delta(f)$, its solution is obtained using the free space Green's Function and has the following form:

$$4\pi\Phi(\mathbf{x},t) = \int_{-\infty}^{t}\int_{-\infty}^{\infty}\frac{Q(\mathbf{y},\tau)\delta(f)\delta(g)}{r}\,\mathrm{d}\mathbf{y}\mathrm{d}\tau \tag{2}$$

in which $f(\mathbf{x},t) = 0$ is the data surface or characteristic cone [3,7]; r is the distance between the observer and the source.

The four dimensional integral in Equation (2) is reduced by using the appropriate variables. In particular, the Retarded Time formulation can be obtained by choosing the following variable representation in Equation (2): $(y_3,\tau) \to (f,g)$. This yields to:

$$4\pi\Phi(\mathbf{x},t) = \int_{f=0}\left[\frac{Q(\mathbf{y},\tau)}{r|1-M_r|}\right]_{\tau^*}\mathrm{d}S \tag{3}$$

All the values in the integral are evaluated at the Retarded Time τ^*:

$$\tau^* = t - \frac{r}{c} \tag{4}$$

Proceeding in the same fashion as above, it is possible to obtain the other formulations which could be already found in Ref. [1] and have been labelled later as Emission Surface (ES) and Collapsing Sphere (CS) formulations in Ref. [7]. From Equation (2) choosing the variable representation: $(y_3,\tau) \to (F,g)$, where $F(\mathbf{x},t,\mathbf{y}) = f(\mathbf{y},\tau^*)$:

$$4\pi\Phi(\mathbf{x},t) = \int_{F=0}\frac{1}{r}\left[\frac{Q(\mathbf{y},\tau)}{\Lambda}\right]_{\tau^*}\mathrm{d}\Sigma \tag{5}$$

where $\Lambda = |\nabla F| = \sqrt{1 - 2M_n\ \cos\ \theta + {M_n}^2}$.

Finally, the CS formulation of equation (2) is obtained using $(y_2,y_3) \to (f,g)$:

$$4\pi\Phi(\mathbf{x},t) = \int_{-\infty}^{t}\int_{f,g=0}\frac{Q(\mathbf{y},\tau)}{r\sin\theta}\,\mathrm{d}\Gamma\mathrm{d}\tau \tag{6}$$

where Γ is the curve intersection between the collapsing sphere and the surface source $f = 0$, and θ is the angle between the unit vector in the normal direction, $\hat{\mathbf{n}}$, and the unit vector in the radiation direction, $\hat{\mathbf{r}}$.

Although the formulations are different, the solution of the retarded time equation is a common factor, even in the CS representation, where τ^* is not visible, Γ is obtained only after solving the intersection between collapsing sphere $g = 0$ and $f = 0$. The steps which compose the base of a noise prediction algorithm, independently from the specific formulation adopted, are:

- For the time t_j and panel δS_i, $i = 1, N_p$, defined by the points (S_{Pi}, $Pi = 1, P_p$) (where S_{Pi} are the number of points chosen to represent every single panel), find the corresponding retarded times τ_{Pi}.
- Calculate the surface area and aeroacoustic integrals over the panel δS_i
- Repeat the calculations until $t_i \le T_E$
- Repeat the process for the total number of panels N_p in which the surface is discretised

The interpretation of the above scheme is straight forward, keeping in mind that $(\mathbf{x},t)$ must be fixed during each step, the complete computation is implemented in three loops. The inner loop evaluates the contribution of all the source δS_i in which the control surface is discretised, for a given $(\mathbf{x}_k,t_j)$, then there is a loop to compute the time history of the sources