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Doubly negative isotropic elastic metamaterial for sub-wavelength focusing: Design and realization

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ABSTRACT

In spite of much progress in elastic metamaterials, tuning the effective density and stiffness to desired values ranging from negatives to large positives is still difficult. In particular, simultaneous realization of double negativity and isotropy, critical in sub-wavelength focusing, is very challenging since anisotropy is usually unavoidable in resonance-based metamaterials. The main difficulty is that there is no established systematic design method for simultaneous achieving of double negativity and isotropy. Thus, we propose a unique elastic metamaterial unit cell with which simultaneous realization can be achieved by an explicit step-by-step approach. The unit cell of the proposed metamaterial can be accurately modeled as an equivalent mass-spring system so that the effective properties can be easily controlled with the design parameters. The actual realization was carried out by acquiring the desired properties in sequential steps which is in detail. The specific application for this study is on sub-wavelength focusing, which will be demonstrated by waves from a single point source focused on a region smaller than half the wavelength. Actual experiments were performed on an aluminum plate where the designed metamaterial flat lens was imbedded. The results acquired through simulations and experiments suggest potential applications of the proposed metamaterial and the systematic design approach in advanced acoustic surgery or non-destructive testing.

1. Introduction

There has been considerable progress in artificial sub-wavelength materials called metamaterials that open up a new way of wave manipulation. Focusing on elastic metamaterials alone, not to mention electromagnetic metamaterials, various interesting wave phenomena have been reported, such as negative density and/or stiffness [1–16], negative refractions [17–20] and cloaking [21–23]. Here, we are concerned with double negative isotropic elastic metamaterials, which have not been studied much due to the difficulty in actual realization. If such metamaterials can be realized and fabricated, they can be

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used to make flat lens that focus wave energy at a focal point smaller than half the wavelength, as sketched in Fig. 1. Certainly, sub-wavelength wave focusing - which is useful in advanced medical imaging, acoustic surgery, non-destructive evaluation, and others - cannot be achieved with natural media due to the well-known diffraction limit.

While there have been a number of studies on sub-wavelength focusing with acoustic waves propagating in fluids [24–32], researches on sub-wavelength focusing with elastic waves propagating in elastic solids are rare. Most of the earlier studies on elastic wave focusing used phononic crystals [33–38], including graded-refractive index phononic crystals. However, it is difficult to achieve focusing at a sub-wavelength scale with phononic crystals. There were also some attempts to focus higher symmetric Lamb wave modes with negative wave speeds in a stepped plate [39,40], but this approach is mainly useful for higher wave modes and cannot be applied to the lowest-order symmetric Lamb wave mode (S0 wave mode), which is perhaps the most useful and widely used wave mode in actual applications. Recently, Bigoni et al. [41] presented simulation results demonstrating a possibility to focus the S0 wave mode (the lowest symmetric Lamb wave mode) by an isotropic negative elastic metamaterial. It appears, however, that their approach may not always guarantee the isotropic elastic metamaterials for a selected frequency, as shown in Ref. [20]. The literature survey above suggests that the elastic metamaterial design for a wide range of applications requires not only an independent control of the effective (negative) stiffness and density, but also the realization of isotropy. Especially for subwavelength focusing, double negative isotropic elastic metamaterials should be designed for any selected target frequency. As such, we present a unique elastic single-phased metamaterial unit cell that can be easily and explicitly tuned to be doubly negative isotropic for any specific frequency.

At this point, it is worth mentioning the difficulty appearing in elastic metamaterials compared to acoustic/electromagnetic metamaterials. We refer to acoustic and elastic waves as waves propagating in fluids and solids, respectively. For acoustic/electromagnetic metamaterials, waves are governed by vector fields. Thus, one can easily achieve two-dimensional isotropic acoustic/electromagnetic metamaterials if the unit cell has the same configuration with respect to the x and y directions. On the other hand, elastic metamaterials are governed by tensor fields, which means that an elastic metamaterial with the same unit cell configuration in the x and y directions does not guarantee isotropy. To explain this issue better, the physics of the in-plane longitudinal wave in an elastic medium is governed by the Christoffel equation [42],

$$1\Big/v_{\mathrm{ph}} = k/\omega = (2\rho)^{1/2}\left(C_{11} + C_{44} + \sqrt{(C_{11} - C_{44})^2 \cos^2 2\phi + (C_{12} + C_{44})^2 \sin^2 2\phi}\right)^{-1/2}. \tag{1}$$

where k, ω, ρ and v_{ph} denote wavenumber, angular frequency, density and (phase) wave speed, respectively, while ϕ is the angle between the wave propagation direction and the x-axis. The stiffness terms are denoted by C_{11} (longitudinal along x_1),

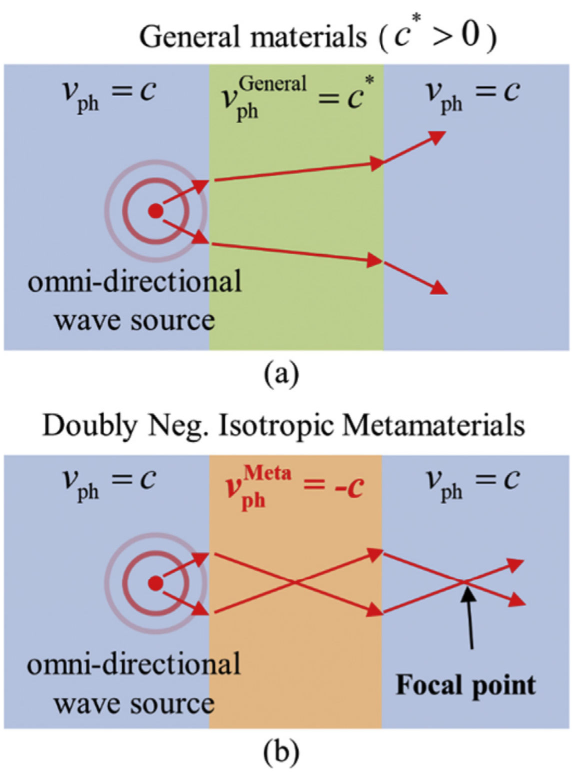

Fig. 1. Wave propagating path through (a) a layer of general material and (b) a layer of a metamaterial having isotropic negative phase velocity (v_{ph}).