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Super-resolution imaging of low-frequency sound sources using a corrected monopole time reversal method

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ABSTRACT

A limitation of the monopole time reversal (MTR) method, which records and retransmits the monopole field only, is that the spatial resolution of the focus cannot be better than half a wavelength, and therefore it is not suitable for locating low-frequency sound sources. In this paper, the time-reversed pressure field obtained by the MTR method is first transformed into the wavenumber domain, and is then decomposed into a filter term that controls the spatial resolution of the focus and a source term that is related to the sound source and focusing plane. Subsequently, a correction is made to the time-reversed pressure field of the MTR method by replacing its filter with the filter for the time-reversed pressure gradient field of the dipole TR (DTR) method, a constant filter and an empirical filter, which makes it possible to include many more evanescent waves and obtain subwavelength focusing. Numerical simulation and experimental results show that compared to the original MTR method, the corrected one is able to dramatically improve the spatial resolution of the focus at low frequencies. It is also found that the constant filter is applicable when the signal-to-noise ratio (SNR) is high (generally above 30 dB), the filter for the time-reversed pressure gradient field of the DTR method works stably even in the situation of low SNR, and the empirical filter performs best when the SNR is above 10 dB.

1. Introduction

The time reversal (TR) method was first introduced by Fink et al. [1,2] to realize the focusing on a reflective target in inhomogeneous media. In the TR method, a signal is recorded by an array of transducers on a closed cavity [3] or on a plane [1,4,5], time reversed and then retransmitted into the medium. The retransmitted signal propagates back through the same medium and refocuses on the source, thus providing the ability to locate the source. So far, the TR method has been widely applied to different fields, such as medicine [6,7], geophysics [8,9], nondestructive testing [10,11], underwater acoustics [12,13], telecommunications [14,15] and audible range acoustics [16–23].

However, the focal spot size of the TR method is limited to half a wavelength even if the initial source dimensions are much smaller than the half wavelength [24]. This limitation makes the TR method less effective in locating sound sources at low frequencies. The main reason for this problem is that subwavelength details that are carried by evanescent waves are lost because the time-reversal mirror is generally located in the far field of the source and the evanescent waves decrease exponentially with the propagation distance and usually do not reach the imaging system [25]. To overcome this limitation,

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some works have been done to improve the resolution of the TR method by introducing or amplifying the evanescent waves in the time-reversed field. Lerosey et al. [26] used a random distribution of subwavelength scatterers placed in the near field of the focusing point to convert evanescent waves into propagating waves and therefore got subwavelength focusing even though the time-reversal mirror was placed in the far field. Rosny et al. [27] proposed to use the acoustic sink in the ultrasonic range to actively cancel the outgoing wave at the source location by adding a TR source term which provides the evanescent waves essentially, and that method was applied successfully in a chaotic environment. Bavu et al. [21] conducted the acoustic sink in a numerical way to overcome the problems of experimental acoustic sink, and they [19,23] also proved that the acoustic sink is effective in a damped and reverberant environment at audible frequencies.

In order to get subwavelength focusing, near-field TR methods also have been developed [25,28,29]. Conti et al. [30] proposed a near-field TR procedure which can provide a high spatial resolution without *a priori* knowledge of the original source by combining the acoustic sink and near-field acoustic holography. The essence of this method is to combine the phase component of the time-reversed field with the amplitude component of the near field of a point source in the wavenumber domain. However, not all TR methods can obtain subwavelength focusing even if the time-reversal mirror is located in the near field of the source. In fact, there are five TR schemes, i.e., the perfect TR (PTR) method that involves a time-reversed version of the Helmholtz-Kirchhoff equation, the monopole TR (MTR) method that records and retransmits only the monopole field, the dipole TR (DTR) method that records and retransmits the dipole field and two mixed-mode TR methods that involve only one of the processes [31]. It has been proved that the time-reversed field of the PTR method contains no evanescent waves even when the time-reversal mirror is very close to the focal spot, while the other four TR methods contain evanescent waves in the time-reversed fields [25,31]. However, only the dipole and mixed-mode TR methods obtain subwavelength focusing and the spatial resolution of the DTR method is much better than those of the mixed-mode TR methods. The MTR method is ineffective in focusing below the half wavelength limit although the time-reversed field contains evanescent waves. The reason is probably that the evanescent waves are insufficient to obtain subwavelength focusing. But from a realistic point of view, the MTR method only needs to record and retransmit the monopole field, and it is more convenient than the DTR method that needs to record and retransmit the dipole field. In addition, compared to near-field acoustic holography [32], the MTR method needs not to calculate the inverse of a matrix and thus avoids the ill-posed problem. Therefore, it is meaningful to improve the performance of the MTR method, and it would be very useful for the localization and identification of noise sources.

Inspired by the work of Conti et al., a corrected MTR (CMTR) method based on pressure measurements on a plane in the near field is proposed. In the method, the time-reversed pressure field obtained by the MTR method is first transformed into the wavenumber domain, and is then decomposed into a term that is related to the source and focusing plane and a term that controls the spatial resolution of the focus, which will be viewed as a filter. Subsequently, the filter for the time-reversed pressure gradient field of the DTR method, a constant filter and an empirical filter are used to replace the original filter for the time-reversed pressure field of the MTR method, and new time-reversed results can be obtained. Because the proposed filters can include many more evanescent waves than the original one, it is possible to obtain subwavelength focusing after this correction.

2. Theory

2.1. The monopole and dipole time reversal methods

In an ideal fluid medium, a point-like source located at $\mathbf{r}_0 = (x_0, y_0, z_0)$ and excited by a waveform $s(t)$ generates a pressure field at $\mathbf{r} = (x, y, z)$, which is the solution of the wave equation with a source term

$$\nabla^2 p_S(\mathbf{r}, t) - \frac{1}{c^2}\frac{\partial^2 p_S(\mathbf{r}, t)}{\partial t^2} = -s(t)\delta(\mathbf{r} - \mathbf{r}_0), \tag{1}$$

where c is the sound speed, δ denotes the Dirac delta function, and ∇^2 represents the Laplacian operator with respect to $\mathbf{r}$. The solution of the pressure field can be given as

$$p_S(\mathbf{r}, t) = s(t) \underset{t}{\otimes} g(\mathbf{r}, \mathbf{r}_0, t), \tag{2}$$

where $g(\mathbf{r}, \mathbf{r}_0, t)$ is the impulse response function in free space, and $\underset{t}{\otimes}$ denotes the time-domain convolution operator. Assume that the pressure field $p_S(\mathbf{r}_m, t)$ is measured at a measurement point $\mathbf{r}_m = (x_m, y_m, z_m)$ during the time interval $[0, T]$. After a TR operation described by the transform $t \rightarrow T - t$, the time-reversed pressure field at the measurement point $\mathbf{r}_m$ is obtained as

$$p_T(\mathbf{r}_m, t) = p_S(\mathbf{r}_m, T - t). \tag{3}$$

Substituting Eq. (2) into Eq. (3) and applying the temporal Fourier transform to Eq. (3), one obtains

$$P_T(\mathbf{r}_m, \omega) = P_S^*(\mathbf{r}_m, \omega)\exp(\mathrm{j}\omega T) = S^*(\omega)G^*(\mathbf{r}_m, \mathbf{r}_0, \omega)\exp(\mathrm{j}\omega T), \tag{4}$$