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Distributed parameter and finite element models for wave propagation in railway contact lines

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ABSTRACT

A distributed parameter model of a railway two-level catenary system is presented for the analysis of the coupled wave dynamics. The wires are modelled as two straight axis parallel beams, with linear equilibrium equations, and the moving load applied by the pantograph is modelled as a constant concentrated travelling force. The general solution is sought by an application of the Ritz–Galerkin method, and then compared with direct time integrations of a finite element model (FEM), achieved by two different integration schemes. The proposed model provides a valid reference for appropriately selecting the FEM parameters, in order to reduce the errors due to spurious modes, affecting the numerical integrations especially at high speeds of the moving pantograph.

1. Introduction

Railway catenary systems, supplying trains with electric power, play an important role in determining the maximum allowable railway velocity, which is limited to a certain percentage of the wave propagation velocity of the contact wire. The analysis of their dynamic behaviour is not straightforward, basically consisting of a wave propagation problem in structures excited by moving loads [1], and the bibliography on the specific topic is not extensive.

In [2] an analytical method is proposed for calculating the steady–state response of a two–level catenary to a moving pantograph, represented by a travelling constant force; the model for the catenary consists of two strings, the upper one fixed at periodically spaced points, connected each other by equidistant lumped mass–spring–dashpot elements. In [3] it is recognized that a beam model with dispersive wave characteristics better represents the contact wire than a string model, and a method is presented in the paper for estimating the wave propagation velocity of a railway contact wire by applying a wavelet transform to experimental signals. The problem of flexural oscillations of a single infinite beam resting on identical periodic simple elastic supports, caused by a harmonic concentrated force moving steadily along the beam, is solved in [4] with an application of the Fourier transform. Using modal analysis, the deflection of a single beam of finite length without intermediate supports, subjected to an axial tensile force and a moving concentrated force, has been analytically determined in [5].

A two–level catenary including bending stiffness in the wires has been considered in [6], expressing the displacements as finite sums of sine functions, and computing the responses of the discretized system via Lagrange equation's method. The

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Nomenclature

General nomenclature

L total length of a single beam (wire) (m)
d diameter (m)
μ mass per unit length (kg/m)
T constant axial load on a single beam (wire) (N)
E Young's modulus (N/m^2)
$I = \pi d^4/64$ area moment of inertia (m^4)
$K = EI$ bending stiffness (Nm^2)
w flexural displacement (m)
$x \in [0, L]$ spatial coordinate
x_j position of the j^{th} intermediate support (registration arm and bracket)
J total number of intermediate supports
x_h position of the h^{th} connection between the wires (dropper)
H total number of droppers
x_F position in which a lumped non-moving force is applied to the contact wire
m_0 equivalent lumped mass of a registration arm (kg)
m_{12} equivalent lumped mass of a dropper, at the connections with the wires (kg)
k_0 linear stiffness of a bracket (N/m)
k_{12} linear stiffness of a dropper (N/m)
P constant modulus of the moving force applied to the contact wire by the pantograph (N)
F constant modulus of a lumped non-moving force applied to the contact wire (N)
v constant travelling speed of the pantograph (m/s)
t time (s)

FEM nomenclature

h time step (s)
l length of the element (m)
$\mathbf{v}_{n+1}$ vector of displacements at time instant $t_{n+1} = h(n+1)$
$\widehat{\mathbf{M}}$ FEM mass matrix
$\widehat{\mathbf{C}}$ FEM damping matrix
$\widehat{\mathbf{K}}$ FEM stiffness matrix
$\widehat{\mathbf{f}}_{n+1}$ FEM forcing vector at time instant t_{n+1}
β Newmark coefficient
γ Newmark coefficient
α_m Generalized-α method averaging coefficient
α_f Generalized-α method averaging coefficient
ρ_∞ Asymptotic spectral radius

same approach has been adopted also in [7], with additional inclusion in the numerical model of a three degrees of freedom pantograph with unilateral contact.

In this study a distributed parameter model of a railway two–level catenary system is presented for the analysis of the coupled wave dynamics. The contact and messenger wires are modelled as two straight axis Euler–Bernoulli parallel beams, while the moving load applied by the pantograph is represented by a constant travelling force. The wires are interconnected by non–equally spaced linearly elastic droppers with lumped masses, and supported by linearly elastic brackets. The contact wire holds lumped masses positioned in correspondence of each bracket and representing the registration arms. The general solution is sought by the Ritz–Galerkin method, using a set of comparison functions [8] given by the eigenfunctions of a pinned–pinned Euler–Bernoulli beam.

The results of the presented method are given providing complete analytical developments and are compared with direct time integrations of a FEM of the system, which is the common approach to solve the problem under analysis [9,10], for its high flexibility and relatively easy implementation.

Unfortunately, the complexity of the dynamic behaviour may cause numerical errors affecting the FEM solutions, especially at high speeds of the moving pantograph. These errors are very often related to numerical wave modes, or spurious modes, introduced by the time integration algorithms.

The most common technique to discard spurious modes consists in adding numerical damping, either by an intrinsic property of the algorithm, as in the Bathe method [11], or by using explicit parameters, as in the Generalized–α method [12]. Both methods are compared with the proposed distributed parameter technique.

2. Distributed parameter model

A distributed parameter model of a railway catenary system is considered, with some simplifying assumptions: the wires are modelled as two straight axis parallel beams, with linear equilibrium equations (neglecting in particular the slackening of the droppers), while the moving load applied by the pantograph is simply modelled as a constant concentrated travelling force. Damping is disregarded in the model under investigation, because it is often considered as negligible (as suggested by [13]). Moreover, one of the main purposes of the paper is to discuss the numerical damping added by integration algorithms and this effect can be better highlighted for undamped systems. Introducing damping phenomena carries further analytical and computational effort, in particular when non-proportional damping distribution is taken into account [14–17].