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Global linear stability analysis of flow in a lined duct

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ABSTRACT

Eigenmodes of the linearised Euler equations are computed in order to study lined flow duct global stability. A simplified configuration is considered and the governing equations are discretised by means of the discontinuous Galerkin method. A biorthogonal technique is used to decompose the global eigenfunctions into local eigenmodes. The system dynamics switches from noise amplifier to resonator as the length of the liner is increased. The global mode in the liner region is shown to be mainly composed of a hydrodynamic unstable wave and a left-running acoustic mode.

1. Introduction

Acoustic lining is widely used to reduce sound. However, for specific liners and under particular flow conditions, some experiments have shown that a convective hydrodynamic instability may grow on the liner and is likely to lead to sound amplification [1–6]. Evidence of such instabilities have been shown as well numerically [7,8].

Hydrodynamic instabilities developing over an infinite acoustic lining have been extensively studied by means of local stability analysis. The first analyses have been made analytically under a constant mean flow assumption by taking into account the resulting vortex sheet on the lining through the Ingard-Myers boundary condition [9,10]. Rienstra [11] showed the existence of two hydrodynamic modes, one of them being potentially convectively unstable according to the so-called "Crighton-Leppington" criterion. However, Brambley [12] showed: firstly, that this criterion is flawed and that the Briggs-Bers criterion has to be used; and, secondly, that the Ingard-Myers boundary condition is mathematically ill-posed since it gives arbitrarily large exponential growth rates for arbitrarily small wavelengths and therefore precludes the Briggs-Bers stability analysis. Two well-posed boundary conditions have then been proposed by Rienstra and Darau [13] and Brambley [14] by considering a thin boundary layer on the acoustic lining. The boundary condition proposed by Brambley [14] was derived by following a more general approach [15] and was shown to give better results [16]. Brambley [17] showed that this boundary condition gives up to six surface modes, one of them potentially being a hydrodynamic convective instability.

Without the constant mean flow assumption, Rienstra and Vilenski [18] observed such a hydrodynamic instability by solving the problem numerically. Moreover, by considering a configuration representative of the experiments by Marx et al. [5], local stability analysis gives a hydrodynamic instability in agreement with the measurements [19–21]. Additionally, a recent study by Khamis and Brambley [22] found that viscosity may dramatically affect stability.

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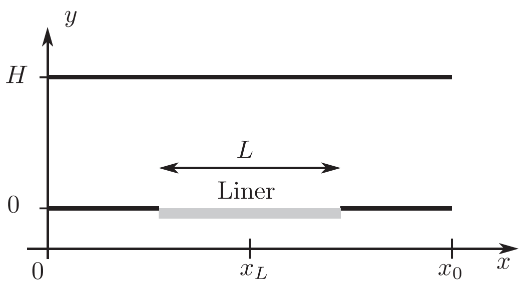

Fig. 1. Geometry of the lined flow duct.

In real flow ducts, the liner is not infinite. Such configurations can be investigated analytically in a local approach by means of the Wiener-Hopf technique. Several studies (see for instance [23]) dealt with this problem but all of them considered mean flows with slipping boundary condition at the liner and used the Ingard-Myers boundary condition. As the problem is non-uniform, a natural alternative is to consider the global modal approach [24,25], where the eigendirections are both the axial and normal directions. Such an approach is followed to characterise resonator dynamics [26] related to the presence of an absolutely unstable region. On the contrary, amplifier dynamics might be investigated by searching for the initial optimal perturbation (see for instance [27] and [28]) or the optimal harmonic forcing [29]. First attempts to account for the finite extent of the liner in modal and non-modal stability approaches have been presented by Pascal et al. [30] (in French) and Pascal et al. [31] respectively. More recently, Rahbari and Scalo [32] performed a global stability analysis of a fully-developed turbulent channel flow with porous wall. They found that the linearised Navier-Stokes equations support a globally unstable mode at a frequency close to the resonant frequency of the liner.

The present paper is intended to apply modal stability analysis to the configuration considered by Pascal et al. [31] in the time domain. This configuration was chosen as a simple toy model to study hydrodynamics instabilities developing in lined ducts. Moreover, the authors have reported [31, Section 3] that this configuration might exhibit a stable or unstable resonator dynamics, depending on the length of the liner. Section 2 introduces the chosen configuration and the governing equations while their discretisation is presented in the following section. Sections 4 and 5 are devoted to the analysis of resonator dynamics by means of modal computations and biorthogonal decomposition.

2. Governing equations and chosen configuration

2.1. Linearised equations

The configuration studied in this paper is a partially lined two-dimensional flow duct of height H. The length of the liner is denoted by L. The geometry is depicted Fig. 1 together with the definition of the coordinates. x_L is the middle of the liner, x_0 the end of the physical domain. The main flow is purely in the axial direction (x-axis) and depends only on y.

The main flow (denoted with subscript 0) is assumed to be subsonic, stationary and homentropic. Moreover, the main density ρ_0 and the speed of sound a_0 are taken as constants[1]. The Mach number of the main flow is denoted M_0, the maximum and mean Mach numbers of mean velocity profile are denoted respectively M and $\bar{M}$. Throughout the rest of this paper, the reference velocity, density, pressure, length and time are respectively $V_{ref} = a_0\bar{M}$, $\rho_{ref} = \rho_0$, $p_{ref} = \rho_{ref}V_{ref}^2$, $l_{ref} = H$ and $t_{ref} = l_{ref}/V_{ref}$.

We are interested by the evolution of a small perturbation, denoted $\boldsymbol{\varphi}$, superimposed to this mean flow. This perturbation is taken under the wave ansatz $\boldsymbol{\varphi}'(x, y, t) = \boldsymbol{\varphi}(x, y)e^{-i\omega t}$ and is composed by the perturbation velocity vector $\boldsymbol{u} = u\boldsymbol{e_x} + v\boldsymbol{e_y}$ and by the perturbation pressure p: $\boldsymbol{\varphi} = (\boldsymbol{u}, p)$. The evolution of this perturbation is governed by the linearised Euler equations in the domain Ω, written under a matrix form as (the Einstein summation is used on x and y):

$$-i\omega\boldsymbol{\varphi} + \boldsymbol{A_j}\partial_j\boldsymbol{\varphi} + \boldsymbol{B\varphi} = 0 \tag{1}$$

where:

$$\boldsymbol{A_x} = \begin{pmatrix} M_0/\bar{M} & 0 & 1 \\ 0 & M_0/\bar{M} & 0 \\ 1 & 0 & M_0/\bar{M} \end{pmatrix}, \boldsymbol{A_y} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \boldsymbol{B} = \begin{pmatrix} 0 & \partial_y\left(M_0/\bar{M}\right) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

[1] For a well-developed parabolic flow, taking a_0 as constant is not rigorously true but is a reasonable assumption.