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Formation of local resonance band gaps in finite acoustic metamaterials: A closed-form transfer function model

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ABSTRACT

The objective of this paper is to use transfer functions to comprehend the formation of band gaps in locally resonant acoustic metamaterials. Identifying a recursive approach for any number of serially arranged locally resonant mass in mass cells, a closed form expression for the transfer function is derived. Analysis of the end-to-end transfer function helps identify the fundamental mechanism for the band gap formation in a finite metamaterial. This mechanism includes (a) repeated complex conjugate zeros located at the natural frequency of the individual local resonators, (b) the presence of two poles which flank the band gap, and (c) the absence of poles in the band-gap. Analysis of the finite cell dynamics are compared to the Bloch-wave analysis of infinitely long metamaterials to confirm the theoretical limits of the band gap estimated by the transfer function modeling. The analysis also explains how the band gap evolves as the number of cells in the metamaterial chain increases and highlights how the response varies depending on the chosen sensing location along the length of the metamaterial. The proposed transfer function approach to compute and evaluate band gaps in locally resonant structures provides a framework for the exploitation of control techniques to modify and tune band gaps in finite metamaterial realizations.

1. Introduction

Acoustic metamaterials (AMMs) are sub-wavelength structures that consist of chains of self-repeating unit cells which house internal elastic resonators. The hallmark feature of AMMs is their ability to realize band gaps, i.e. regions of blocked wave propagation, in low frequency regimes. Band gaps in AMMs primarily depend on the resonator properties and are, thus, size-independent and mechanically tunable [1]. Unique wave propagation behavior in AMMs have rendered them appealing for a wide range of damping and noise control applications. Over the past few decades, AMMs have been investigated in the context of discrete lumped mass systems [2,3], elastic bars [4,5], flexural beams [6–14], as well as 2D membranes and plates [15–18]. Given the dependence of the band structure of the AMM unit cell on resonator parameters, multiple efforts have also been placed on piezoelectric, or actively controlled, metamaterials [19–23].

Acoustic metamaterials are most commonly modeled using a Bloch-wave propagation model of the self-repeating unit cell with periodic boundary conditions [24–28]. Wave propagation methods assume traveling wave propagation in an infinitely-long metamaterial comprised of the individual unit cells. The occurrence of band gaps in these infinite structures

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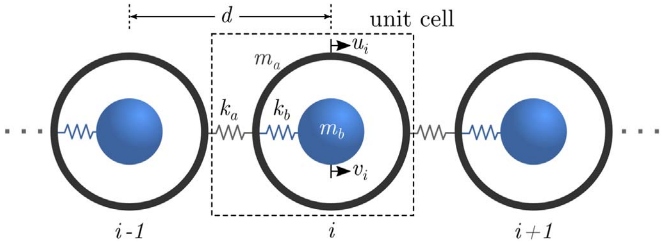

Fig. 1. A lumped mass-in-mass locally resonant acoustic metamaterial.

has been explained in light of gaps in the unit cell's dispersion curve (band diagram) and/or the negative effective mass density concept [29,30]. Discrepancies in the response of actual metamaterials motivated several efforts to understand band gap realizations in finite structures and the effect of imposed boundary conditions [31–33]. Significant among those is the investigation of the relationship between the borders of Bragg-effect band gaps in phononic (periodic) structures and the corresponding eigenfrequencies, explained using the phase-closure principle [32,33]. Modal analysis has also been utilized to develop a mathematical formulation to estimate locally resonant band gaps and provide design guidelines and insights into the choice of resonators and their optimal locations [34]. To this date, however, a mathematical framework that explains and quantifies the evolution of local resonance band gaps in finite AMMs remains lacking.

In this effort, we focus on AMMs where the number of cells, as well excitation and response locations, are specified. We derive a generalized dynamic model to evaluate the input-to-output transfer function associated with such locally resonant structures, and explain the formation mechanism of the band gap in light of their frequency response and pole-zero (PZ) distributions. To facilitate the discussion and advance a closed-form solution, the analysis is carried out on a one-dimensional mass-in-mass type metamaterial. The AMM consists of a chain of spring-mass unit cells shown in Fig. 1. In the presented analysis, damping elements are excluded from both the base and the local structure for two important reasons: (1) to neutralize the effect of dissipation on the band gaps, an effect that has been recently investigated in a number of efforts [35–38], and (2) to ensure that any damping captured in the numerically computed poles or zeros in the lengthy expressions of the developed dynamic model of the finite AMM are merely a result of minor errors or computational precision, as will be highlighted later in the discussion. The limiting case of the presented approach as the length of the AMM chain approaches infinity matches the traditional Bloch-wave analysis and bridges the gap between the two approaches.

Finally, the discussion is extended to explain and differentiate between the effects of sensing location (i.e., location where displacement is measured) and the effect of the number of cells on the bandwidth and degree of attenuation obtained from the local resonators. Analysis of AMMs from a dynamic systems perspective provides a physical insight into the formation of these band gaps over a specific range of frequencies, and provides a clear distinction between the operation concepts of AMMs and tuned dynamic absorbers from a vibrations standpoint. Furthermore, explaining the behavior in terms of frequency domain tools and PZ maps sets a future framework for implementing control techniques. Finally, the investigation of finite metamaterial structures is naturally of interest since the results directly impact the fabrication of realistic and physically realizable, rather than purely theoretical, AMMs.

2. Dynamics of 1-D acoustic metamaterials

2.1. Wave dispersion analysis

The simplest example of an AMM is a periodic series of spring-mass systems hosting internal spring-mass resonators, as shown in Fig. 1. For an AMM with cell spacing d, identical outer masses m_a and inner masses m_b connected via springs k_a and k_b, the governing motion equations for the i^{th} unit can be derived as:

$$\begin{bmatrix} m_a & 0 \\ 0 & m_b \end{bmatrix}\begin{Bmatrix} \ddot{u}_i \\ \ddot{v}_i \end{Bmatrix} + \begin{bmatrix} 2k_a + k_b & -k_b \\ -k_b & k_b \end{bmatrix}\begin{Bmatrix} u_i \\ v_i \end{Bmatrix} + \begin{bmatrix} -k_a & -k_a \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} u_{i-1} \\ u_{i+1} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{1}$$

where u_i and v_i represent the displacements of m_a and m_b of the i^{th} cell, respectively. By applying the harmonic wave solution to the above motion equations, the dispersion relation can be derived as [3]

$$A_1\omega^4 + A_2\omega^2 + A_3 = 0 \tag{2}$$

with

$$A_1 = m_a m_b, \quad A_2 = -\left[(m_a + m_b)k_b + 2m_b k_a(1 - \cos\bar{\beta})\right], \quad A_3 = 2k_a k_b(1 - \cos\bar{\beta})$$