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Acoustical source reconstruction from non-synchronous sequential measurements by Fast Iterative Shrinkage Thresholding Algorithm

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A B S T R A C T

Acoustical source reconstruction is a typical inverse problem, whose minimum frequency of reconstruction hinges on the size of the array and maximum frequency depends on the spacing distance between the microphones. For the sake of enlarging the frequency of reconstruction and reducing the cost of an acquisition system, Cyclic Projection (CP), a method of sequential measurements without reference, was recently investigated (JSV,2016,372:31-49). In this paper, the Propagation based Fast Iterative Shrinkage Thresholding Algorithm (Propagation-FISTA) is introduced, which improves CP in two aspects: (1) the number of acoustic sources is no longer needed and the only making assumption is that of a “weakly sparse” eigenvalue spectrum; (2) the construction of the spatial basis is much easier and adaptive to practical scenarios of acoustical measurements benefiting from the introduction of propagation based spatial basis. The proposed Propagation-FISTA is first investigated with different simulations and experimental setups and is next illustrated with an industrial case.

1. Introduction

Acoustic reconstruction consists in measuring the acoustic quantity in the field by a sensor array and reconstructing the image of noise sources by back-propagation algorithms (inverse problem). It has a wide range of applications in source identification, vibration analysis and machine diagnosis [1–3]. A fundamental limitation of acoustic source reconstruction is imposed by geometry of the array. Specifically, the minimum frequency of reconstruction depends on the dimension of the array and the maximum frequency depends on the distance between neighboring microphones [4]. A solution to achieve a large array and/or high microphone density is to scan the object of interest by moving sequentially a prototype array (an arbitrary array of achieving the sequential measurements), which is referred to as sequential measurements [5,6]. In order to synchronize the measurements taken at different times, a set of fixed reference microphones are located in the acoustical field, which are fully correlated with the sources so as to keep track of phase relationships of adjacent measured in consecutive positions of the array.

However, it generally accepted that the provision of reference microphones incurs an extra cost and precludes some microphones to be used when the user has to face a limited number of tracks in the acquisition system. The referenced

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method actually requires that the number of references to be larger than the number of independent sources, therefore, the determination the number of independent sources is a thorny problem in practice. Last but not least, choosing location of the reference microphone can be also problematic in some scenarios.

Much research in recent years has focused on the sequential measurements without reference. For instance, sequential measurements from intensity measurements (i.e.,Broadband acoustic holography from intensity measurements (BAHIM)) [7–9] breaks through the restriction of using references by reconstructing the phase from the quadratic pressure and tangential components of the sound intensity, yet it requires the use of an array of intensity probe.

Sequential measurements from pressure measurements without reference has been firstly addressed in Ref [10] by taking the incomplete measurements explicitly into account to formulate the acoustic inverse problem. It has been next investigated in Ref [11], which is reformulated as a matrix completion problem, and the Cyclic Projection (CP) algorithm was proposed correspondingly to find a full spectral matrix subject to given constraint of hermitian symmetry, measurement fitting, reduced rank and spatial continuity of the sound field. However, there are two difficulties to apply the CP algorithm: one is that the number of acoustical sources should be known (or at least to be estimated) a priori, the other is that the smoothness condition was achieved by projection with a somewhat arbitrary spatial basis, therefore, without guiding principle about how to construct this basis. In this paper, a method that is different from [11] is proposed, with two main contributions as which are highlighted hereafter.

- Weakly sparse assumption of the eigenvalue spectrum. The eigenvalue spectrum of the spectral matrix is assumed to be weakly sparse (full rank but with only a few dominant eigenvalues), which corresponds to a small value of the nuclear norm (i.e. of the sum of the eigenvalues). Not only is this assumption more general than the low rank assumption of Ref [11], but it also avoids the difficulty of a priori determining the actual rank of the spectral matrix (number of sources). On this basis, a global solution is proposed that minimizing the nuclear norm (convex cost function).
- Propagation based spatial basis. A new spatial interpolation basis is obtained from the singular value decomposition (SVD) of the propagation functions between the acoustical sources and non-synchronous sequential measurements is applied, which is considered being adaptive to practical scenarios of acoustical measurements. It is significant that this basis is easy to construct based on a “rule of the thumb” as in Ref [10,12].

It is worthy to note that the proposed method in this paper also has potential application to further improve the Average Beamforming method in Ref [13] and 3D sources imaging. The paper is organized as follows. In Section 2, the sequential measurements problem is firstly defined and formulated as a completion problem of the spectral matrix. The weakly sparse assumption of the eigenvalue spectrum and corresponding Fast iterative shrinkage thresholding algorithm (FISTA) is introduced in Section 3. A comparison between CP and FISTA is drawn in Section 4. Sections 5 and 6 deal with parametric analyses and experimental validation. An industrial application is finally given in Section 7.

2. Spectral matrix completion problem in microphone array sequential measurements

Let $s(\mathbf{r}, \omega; \zeta)$, $\mathbf{r} \in \Gamma$ be the source distribution of interest (eg. normal velocity or acoustic pressure) on the source surface Γ in front of (or enclosing) the radiating object D, at a given frequency ω. The fundamental premise is that the acoustic source distribution is considered as a stationary stochastic field (only the temporal stationarity is assumed), thus the acoustic sources produce outcomes whose realizations depend on the events ζ in sample space Ω. From a practical point of view, an outcome of the source distribution will simply correspond to a snapshot of the measured signal (i.e. the Fourier transform of a short-time segment possibly tapered with a smooth data window) at a given position of the array. The acoustic field that is produced by the radiating body D is measured at some discrete locations $r_{m,i}$, where $r_{m,i}$ denotes the position of m-th microphone in the array at the i-th position of the array, where $m = 1, \ldots, M$, $i = 1, \ldots, P$, M is the number of microphones in an array and P the measurements times. Assume that $p(r_{m,i}, \omega; \zeta)$ is the acoustic pressure measured at the position of $r_{m,i}$ (the positions are shown in Fig. 1), which denotes the short time Fourier transform of the pressure signal from the m-th channel of the array at i-th position with frequency ω and datum ζ [14–16]. It is denoted that $\hat{\mathbf{S}}_{pp}^{m}$ and $\hat{\mathbf{S}}_{pp}$ are respectively the sequential measurements spectral matrix and simultaneous measurement spectral matrix, which are calculated from the pressure measurements of array (a detailed description of the forward acoustical model and the calculation of spectral matrix can be found in Ref. [11]).

An essential question to be addressed now is the difference between sequential measurements and simultaneous measurement from the spectral matrix perspective in the acoustical inverse problem. By sequential measurements, each measurement of prototype array can obtain all the correlation information between the measurements of microphones, thus all spectral sub-matrices $\hat{\mathbf{S}}_{pp}^{(i)}$, $i = 1, \ldots, P$ (each with size $M \times M$, see Fig. 1) are full matrices. An incomplete spectral matrix $\hat{\mathbf{S}}_{pp}^{m}$ is constructed by rearranging all spectral sub-matrices $\hat{\mathbf{S}}_{pp}^{(i)}$ methodically in block diagonal positions and the remaining positions are padded by zero elements (see left graph in Fig. 2 and note that zero padding is just a way to visualize the measured matrix, it is in fact an unknown part). Contrary to sequential measurements, $\hat{\mathbf{S}}_{pp}$ is a full matrix since