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Analysis of the effect of a rectangular cavity resonator on acoustic wave transmission in a waveguide

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ABSTRACT

The transmission of acoustic waves along a two-dimensional waveguide which is coupled through an opening in its wall to a rectangular cavity resonator is considered. The resonator acts as a classical band-stop filter, significantly reducing acoustic transmission across a range of frequencies. Assuming wave frequencies below the first waveguide cut-off, the solution for the reflected and transmitted wave amplitudes is formulated exactly within the framework of inviscid linear acoustics. The main aim of the paper is to develop an approximation in closed form for reflected and transmitted amplitudes when the gap in the thin wall separating the waveguide and the cavity resonator is assumed to be small. This approximation is shown to accurately capture the effect of all cavities resonances, not just the fundamental Helmholtz resonance. It is envisaged this formula (and more generally the mathematical approach adopted) could be used in the development of acoustic metamaterial devices containing resonator arrays.

1. Introduction

The Helmholtz resonator is a well-known acoustical device in which a volume of air inside a rigid vessel is made to resonate by exciting acoustic oscillations at its mouth. The original formula for the fundamental resonant frequency ω_h due to Helmholtz was later generalised by Rayleigh [1] and can be expressed as

$$\omega_h \approx c_s\sqrt{\frac{S}{VL'}} \tag{1.1}$$

where c_s is the wave speed in the acoustic medium, S is the area of the mouth which is assumed to be attached to the resonator body of volume V through a neck of length L. Here $L' = L + l$ is an effective neck length which takes account of added inertia effects and is dependent on the geometry of the neck (often determined semi-empirically and proportional to $S^{1/2}$). The formula above (also see Kinsler et al. [2], §10.8 or Chanaud [3]) is approximate, based on the long-wavelength assumption: $\lambda \gg (L', S^{1/2}, V^{1/3})$. It assumes the mass of air in the neck acts mechanically as an incompressible piston connecting the oscillatory pressure at the mouth to the compressible volume of trapped air in the vessel, which in turn acts as a spring.

When a Helmholtz resonator is connected to the wall of a pipe along which acoustic waves are propagating the combined effect can be to drastically alter the acoustic output from total to zero acoustic transmission. This effect is well known

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and has been exploited, for example, by the automotive industry in engine exhaust systems to suppress noise or improve engine performance. For example, Kinsler et al. [2], §10.11 and Chen et al. [4] derive the following formula for the coefficient of transmitted power, $|T|^2$, for a wave of frequency ω propagating along a pipe of cross sectional area $A \ll \lambda$ attached to a Helmholtz resonator:

$$|T|^2 \approx \frac{1}{1 + \left(\dfrac{c_s/2A}{\omega L'/S - c_s^2/\omega V} \right)^2}. \tag{1.2}$$

(This formula is also derived on a long wavelength assumption, ignoring the diffractive effect of the relatively small mouth of the resonator.) It shows that there is a significant reduction in acoustic transmission over a broad range of frequencies around $\omega = \omega_h$ where $|T| = 0$. On account of the analogy with mechanical systems used to develop (1.2), there also exists an analogy with electronic circuitry where the effect of Helmholtz resonators can be reproduced with inductors and capacitors to form a band-stop filter (e.g. Montgomery et al. [5]).

Helmholtz resonators are used in many applications beyond those already mentioned above, for example in quantum, microwave and optical waveguides (e.g. Shao et al. [6], Xu et al. [7] and Scharstein [8]). In the theory of water waves, flat-bottomed harbours with small entrances form Helmholtz resonators. This gives rise to the so-called "harbour paradox" – see Mei [9] who considered a rectangular harbour connected to a semi-infinite ocean through a small gap in a thin wall – in which the smaller the entrance to the harbour the stronger the resonant effect within it.

More recently, Helmholtz resonators have been used extensively in the development of so-called metamaterials and metasurfaces. Thus arrays of sub-wavelength cavities can produce surprising effects upon the macroscopic wave field that are not manifested in naturally-occurring materials. For examples, see Richoux and Pagneaux [10], Fang et al. [11], Wang et al. [12], Seo et al. [13] and Faure et al. [14].

The formulae produced in (1.1) and (1.2) above are approximate and are presumably sufficiently accurate for many applications. However, as highlighted by Chanaud [3], they are only appropriate under a long-wavelength assumption and neglect the effects of higher resonant frequencies. The work in this paper – set in the context of two-dimensional acoustics – is aimed at producing an accurate prediction of the transmitted acoustic wave energy in closed form based upon the solutions of the exact equations of linearised acoustics and without making a long-wavelength assumption from the outset. The need to such resolve higher resonant frequencies and accurately encode their effects in scattering coefficients has recently been highlighted in an sound absorption applications of Romero-García et al. [15] and Jiménez et al. [16].

In the particular problem considered here an incident acoustic wave propagates along a uniform waveguide and interacts with a rectangular cavity through an opening in the waveguide wall. The wall between the waveguide and the cavity is assumed thin so there is no length L assigned to the neck of the resonator. In Section 2 the solution to the problem posed is formulated in terms of integral equations. Solutions are expressed in terms of a series of prescribed functions as a practical means of determining numerical solutions to the integral equations. This forms the basis of the approximate solution for a small gap which is described in Section 3 and relies on some complicated technical details contained in Appendices A and B. The approach here has some similarities with recent work of the authors (see Evans & Porter [17]) in a related problem involving approximating the effect of small gaps on waves. It also shares similarities with the approach taken by Scharstein [8] in a related problem in which the cavity is excited by plane waves from a semi-infinite domain. An alternative approach here could have been to use the method of matched asymptotic expansions as in Mei [9] by connecting solutions to an inner problem in the vicinity of the gap to an outer solution in which the gap acts as a point source. The outcome of the two approaches have much in common although our central positioning of the gap within the cavity introduces difficulties avoided by Mei [9] who considered only off-centre gaps. Recent unpublished work (Prof. David Abrahams, personal communication) has followed this approach.

By using our new approximation for small gaps in the long wavelength limit we shall also be able to derive an explicit expression for L' (or l) in (1.1) for the geometry under consideration. The result is not a simple linear scaling with gap size as is commonly assumed. In addition to making the connection with the Helmholtz resonance, Section 4 describes the effect that higher-order cavity resonances have on $|T|$. In Section 5 we present numerical results which test the new approximation against computations based on the exact formulation. Finally in Section 6 we summarise the paper and suggest how this work could be used elsewhere.

2. Exact treatment of the problem

2.1. Formulation

An infinitely-long waveguide has parallel acoustically-hard walls along $y = 0$ and $y = 1$ for $-\infty < x < \infty$. A small gap in the wall $y = 1$ extends from $x = -a$ to $x = a$ and connects the waveguide symmetrically to a rectangular basin or cavity of width $2b$ and height c (see Fig. 1). All lengths are considered dimensionless, having being scaled by the channel width. The acoustic pressure is written $\Re\{\phi(x, y)e^{-i\omega t}\}$ having angular frequency ω and $\phi(x, y)$ satisfies the wave equation