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Analysis of the external radiation from circular cylindrical shells

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ABSTRACT

Structurally radiated sound power is a critical design parameter. The acoustic radiation mode approach for computing sound power was developed in the early 1990s and has since been widely used. It has been shown to be a rather efficient approach for determining the radiated sound power. In previous research, radiation mode expressions have been developed for planar and spherical structures, as well as axisymmetric modes of internal and external radiation from cylinders. In this work, the radiation modes for external radiating cylinders which account for both axial and circumferential dependence are presented. The expressions are uniquely developed using cylindrical basis functions which are a more natural match to the geometry than past developments, which have been based on spherical harmonics. Higher order radiation modes than have been previously presented are shown. The “leapfrog effect”, whereby higher order modes leapfrog over lower modes in terms of their radiation efficiencies as the frequency goes above the cut-on frequency for those modes, is discussed in detail. The relationships between the mode efficiency and the coincidence effect associated with the cut-on frequencies of the vibration modes are described.

1. Introduction

It is often desirable to know the level of sound radiated from a structure and to understand the structural response that causes the radiation. A suitable metric for quantifying the sound field is the radiated sound power. There are generally two analytical methods for calculating the radiated sound power. The first method utilizes the amplitudes of the structural modes through use of the power transfer matrix, and is computationally expensive to implement [1,2]. This is because the use of structural modes to calculate sound power results in a double surface integral (or sum) over the modes that contribute. In addition, when using the structural modes to determine the sound power, typically many modes are required in the model to ensure convergence. It is not unusual for hundreds of modes, to be included in the summation to ensure convergence.

The second method utilizes the decomposition of the sound field into orthogonal acoustic basis functions, called acoustic radiation modes, by discretizing the vibrating structure as a set of elementary radiators, and incorporating this discretization into the radiation resistance matrix [3,4]. These acoustic radiation modes are a different set of orthogonal modes that are

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naturally suited to efficient decomposition of the acoustic field. In comparison with the first method, this method of using the radiation resistance matrix is typically less intensive to implement [1,2]. This is because the use of acoustic radiation modes to calculate sound power only requires a single surface integral (or sum) over the modes that contribute. In addition, because these modes are naturally suited for the acoustic problem, the modal amplitudes typically drop off rapidly after a relatively small number of contributing modes. This makes the acoustic radiation mode approach much more efficient for calculating radiated sound power.

In 1990, Borgiotti suggested using a modal expansion for representing the radiated sound field. He introduced the concept of the radiation operator, which operates on the structural velocity and yields the far-field radiated pressure [3]. The eigenvectors of this operator create a set of basis functions associated with the radiated sound power that are known as the *radiation modes*. Each eigenvector describes the independent velocity distribution for that radiation mode. The eigenvalues of the radiation operator describe the radiation efficiency of each eigenvector, and describe how efficiently each velocity distribution is able to couple with the surrounding medium and contribute to the radiated sound power. This approach was a significant departure from traditional structural approaches, in that the radiation is described in terms of an orthogonal decomposition of the acoustic field (radiation modes) rather than utilizing the orthogonal decomposition of the structural field that results in structural modes. One result of this approach is that the acoustic field can generally be characterized by a significantly smaller number of radiation modes, whereas a structural mode approach generally results in the need for retaining a very large number of structural modes.

Borgiotti introduced the concept of spatial and radiation filtering to decompose the velocity of the structure into radiating and non-radiating components [5]. As an example, Borgiotti solved the radiation problem numerically using the radiation operator for external radiation from a finite cylinder with flat endcaps, using a Fourier series. He then presented the eigenvectors of this operator as singular velocity patterns and radiation patterns for the circumferentially symmetric zero-order modes [3,5].

About the same time, in 1990, Photiadis introduced singular value decomposition (SVD) as a tool to perform the eigenvalue analysis for acoustic radiation problems [6]. He solved the radiation problem for external radiation from a finite cylindrical shell with hemispherical end caps. He was able to present the eigenfunctions in the wavenumber domain, but again only for the zero-order modes. Bringing the concept of efficiencies (eigenvalues) into the wavenumber domain, Photiadis addressed the effect of wavenumber filtering in terms of coincidence, as well as being able to identify supersonic and subsonic components.

In the same year, Sarkissian developed a new operator, called the radiation resistance operator, which provided a more efficient means of calculating radiation modes [4]. Using spherical harmonics as the set of basis functions for the radiation problem, she developed the general case of the radiation resistance operator, using a numerical boundary element method. She presented some of the zero-order radiation modes (source modes) at a single frequency for radiation from a finite cylinder with flat endcaps, as well as some of the eigenvalues as a function of the spherical harmonics index.

In 1995, Chen and Ginsberg used the reciprocity properties between pressure and velocity to analyze the radiation resistance of structures and presented some numerical results for radiation modes of a spheroidal body [7]. In 1998, Naghshineh and Koopmann used a wave superposition technique to estimate the interior sound field inside a cylindrical shell with endcaps. Using the wave superposition method, they used a limited number of point sources inside the shell to generate the interior acoustic field rather than the large number of monopoles utilized by the boundary element method. They were able to present analytical expressions for the elements of the radiation resistance matrix for the interior radiation of the shell [8,9].

In 1998, Cazzolato and Hansen obtained the radiation modes for the interior sound field in a cylindrical shell by solving the eigenvalue problem for the acoustic potential energy rather than the radiated sound power, using numerical methods. They tried to apply active noise control to the radiated sound inside the shell by sensing the radiation modes obtained from the acoustic potential energy [10].

Radiation modes also provide a means to better understand the sound radiated from a vibrating structure, which can be very beneficial. For instance, radiation modes can be utilized for active structural acoustic control (ASAC) purposes. Knowing the radiation modes for a certain structure allows one to target the most efficient radiation modes as a means of attenuating the radiated sound power [2,10,11]. They can also be used as spatial and radiation filters in active control [5]. Another approach that utilizes radiation modes for ASAC would be to force the structure to couple most efficiently with weak radiators, in order to reduce the radiated sound [12,13].

In this paper, the case of external radiation from a cylindrical shell in an infinite baffle has been investigated and an analytical solution has been developed for the radiation resistance operator for external radiation. Due to the geometry of the shell, Fourier basis functions have been used rather than spherical harmonics. Although the shapes of the radiation modes bear a resemblance to the structural modes, they are fundamentally different in that 1) they do not satisfy the structural boundary conditions, and 2) they result from an orthogonal decomposition of the acoustic field, rather than the structural response. Radiation modes for external radiation from the cylindrical shell are presented for higher non-symmetric modes that represent both axial and circumferential dependence. The results illustrate two important features of the external radiation modes that have not been reported in the literature: 1) the frequency dependence of the radiation modes, including the appearance of the “leapfrog effect”, and 2) the grouping of eigenvalues, both of which will be discussed in the following sections.

In the remainder of this paper, a brief review of radiated sound power is provided to motivate the concept of the radiation resistance operator. With this background, an analytical solution is developed to yield the radiation resistance